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Pump-Leak Models of Cell Volume and Electrolyte Control

Homeostatic control of cell volume and intracellular electrolyte content is a fundamental problem in physiology and is central to the functioning of epithelial systems. These physiological processes are typically modeled using pump-leak models, a system of differential algebraic equations that describes the balance of ions and water flowing across the cell membrane. Despite their widespread use, very little is known about their mathematical properties. In this talk, we present recently established analytical results on the existence and stability of steady states for a general class of pump-leak models. We treat two cases. When the ion channel currents have a linear current-voltage relationship, we show that the there is at most one steady state, and that the steady state is always asymptotically stable. When minimal assumptions are placed on the properties of ion channel currents, we show that there is an asymptotically stable steady state so long as the pump current is not too large. The key tool in our argument is a free energy relation satisfied by a general class of pump-leak models, which can be used as a Lyapunov function to study stability. It time permits, we will discuss spatial generalizations of the pump-leak models and present some preliminary applications of the model.