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Symmetries, conservation laws, Lagrangians and ... quantization of biological systems?

In a very interesting recent review to honor the 50th Anniversary Year of the Journal of Theoretical Biology one reads: *It is frequently claimed that like Newton's invention of calculus biological theory will require 'new mathematics'.... There are, however, many areas of mathematics that have been neglected by theoretical biology that could prove to be of great value. Einstein's work on general relativity, for instance, made good use of mathematical ideas, in particular differential geometry that had previously been developed with completely different motivation. More likely than not, the formal structures have been set forth in some context, and await their discovery and subsequent development in representing biological theory* [1].

Since many mathematical tools used in Physics have also been used in biology with alternate success, we present a somewhat forgotten and neglected tool, a tool that in one of its outcomes, Noether symmetries, helped Einstein and Klein in their quarrel with Hilbert about the energy-momentum conservation of general relativity theory [2]. This tool is Lie continuous symmetries, that yield conservation laws, calculus of variation setting, and ultimately quantization.

The application of Lie symmetries to various biological models have already been shown to either provide more accurate predictions [3] or implement [4], [5], [6] the usual techniques related to qualitative and numerical analysis, that are common tools for any mathematical biologist.

We would like to stir up some controversy with the purpose of making both mathematicians and biologists pondering over some missed opportunities [7].

Since a good example is the best sermon, classical known mathematical models such as the Volterra-Verhulst-Pearl equation [8], [9] shall be used to show the many symmetries and conservation laws they possess, the many Lagrangians and therefore different variational problems they admit [10], and finally the quantization (through Schrödinger equation) that they lead to.

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