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A mathematical model of *Daphnia* epidemics: how resources and competitors alter the disease dynamics

Daphnia dentifera are small herbivorous crustaceans living in freshwater lakes and feeding on algae. They are being preyed upon by fish and other predators such as the phantom midge *Chaoborus* and they can become infected by the virulent fungus *Metschnikowia bicuspidata*. Once infected there is no recovery so *Daphnia dentifera* die releasing fungal spores. In many Midwestern lakes they can also compete for resources with *Daphnia pulicaria* which do not become infected by *Metschnikowia bicuspidata*.

Taking all the interactions into account we obtained a model with five populations: healthy *Daphnia dentifera*, infected *Daphnia dentifera*, fungal spores, algae, and *Daphnia pulicaria*. Specifically, we assumed that the *Daphnia* birth rates are nonlinearly dependent on the algae, background death and predation rates are constant and infectivity depends nonlinearly in the algae and the fungal spores. Infected hosts die at a higher rate due to the disease and because they are being preferentially preyed upon. The fungal spores released per infected hosts that die depend on the host growth rate.

The analysis of the model reveals multiple positive equilibria that undergo a wide variety of bifurcations. In the host – algae system we found an interior equilibrium that exhibits a supercritical Hopf bifurcation as the maximum feeding rate of the host increases. In the host – algae – inferior competitor system oscillations of the host and algae develop, while the competitor dies out. We also found a parameter domain of coexistence via a limit cycle. In the absence of the competitor, the system exhibits damped oscillations and disease is stabilizing. Finally, in the five-dimensional system friendly competition is observed which results in damped oscillations. Hence one might use the ’disease dilution’ by inferior competitors as a strategy to control disease.