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From diffusion models to patch models using mean occupancy time.

Patch models, coupled systems of ODEs, have been widely used by ecologists to study population dynamics on heterogeneous landscapes. The landscape is viewed as a system of coupled homogeneous patches with migration between these patches. This simplification typically looses any explicit connection between migration and individual level movement rules on the landscape, but has the advantage that it is relatively simple to analyse. In contrast, diffusion models can include explicit mechanistic movement rules for a population, but have the disadvantage of being difficult study on complex landscapes. Our approach is to study questions of population persistence on heterogeneous landscapes by deriving an ODE patch model approximation of a corresponding diffusion model which retains information about the underlying movement rules. To do this we introduce the idea of mean occupancy time (MOT), the time an individual spends in a given region. We use MOT to approximate the eigenvalues of the diffusion models which in turn allows us to obtain migration rates for our patch model. We demonstrate that the MOT patch model accurately approximates the persistence conditions of the underlying PDE models in a range of examples. Moreover the approach can also be used to estimate the spatial distribution of a population at equilibrium.