Stochastic Models for Competing Species with a Shared Pathogen

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Abstract: We study the dynamics of deterministic and stochastic models for n competing species with a shared pathogen. The deterministic model is a system of ordinary differential equations for n competing species in which a single shared pathogen is transmitted among the n species. There is no immunity from infection, individuals either die or recover and become immediately susceptible, an SIS disease model. Analytical results about pathogen persistence or extinction are summarized for the deterministic model for two and three species and new results about stability of the infection-free state and invasion by one species of a system of n-1 species are obtained. New stochastic models are derived in the form of continuous-time Markov chains and stochastic differential equations. Branching process theory is applied to the continuous-time Markov chain model to estimate probabilities for pathogen extinction or species invasion. Finally, numerical simulations are conducted to explore the effect of disease on two-species competition, to illustrate some of the analytical results and to highlight some of the differences in the stochastic and deterministic models.