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Theoretical Study of Algal Bloom Dynamics with Akinete Formation and Germination

The aim of this study is to investigate the population dynamics of harmful algal bloom by using the Joehnk's model with some generalizations. The present mathematical model is a second-order nonlinear nonautonomous system based on delay differential equations. One variable is the biovolume of vegetative cells and the other one is the number of units of akinetes. The akinete is a thick-walled dormant cell derived from a vegetative cell and it is considered as a potential and significant factor triggering the algal bloom. The formation and germination of akinetes are modeled with time delays, and the coefficients in the model are described by piecewise continuous functions in time. The time delay related to the carrying capacity is considered in the form of the Hutchinson's equation. The analysis is divided into two parts. In the first part, we investigate the local and global stability of an approximated autonomous system. In the second part, we discuss the numerical solutions of the original nonautonomous system for various simulation conditions.

In the first part, the local stability at critical point is analyzed for an approximated autonomous system derived from averaging the time-varying periodic coefficients. The Jacobian matrix obtained from the linearization near the critical points shows that one critical point at the origin is a saddle point. In the first quadrant where both populations are positive, the saddle point plays a role of unstable node because the eigenvector corresponding to the negative eigenvalue lies in the second and the fourth quadrant in the phase plane. The other critical point located in the first quadrant is an asymptotically stable node. There is no periodic solution in the autonomous system. By calculating the trajectories for various initial conditions and constructing a Lyapunov function, the asymptotically stable node is identified by a globally asymptotically stable node in the first quadrant. Nullclines, critical points and direction fields of the present system are compared with the similar but different autonomous systems such the prey-predator model and the Lotka-Volterra competition model.

Since the original nonautonomous system is highly complicated for mathematical analysis, in the second part, we performed the numerical simulation by solving the delay differential equations with the use of Runge-Kutta algorithm. From the simulated results in cases of virtual environments, we observed the following properties of the model: (i) the initial populations of vegetative cells and akinetes have no influence on the stationary periodic solutions, (ii) temperature variations are closely related to the population dynamics of vegetative cells and akinetes, especially to the number of blooming peaks, (iii) time delays related to the germination and formation of akinetes were negligible in population dynamics whereas the time delay related to the carrying capacity is strongly influential to the populations of vegetative cells and akinetes as well as the occurrence of an additional blooming peak.