Optimal reaction norm for varying environmental states

It is sometimes observed that organisms alter their own phenotype in response to environmental variation without changing their own genotypes. This is called "phenotypic plasticity". This is referred to as "adaptive phenotypic plasticity" especially when the phenotypic change is an advantageous response. The plasticity can be represented as a series of phenotypic values on an axis of environmental states of which functional form is called the "reaction norm". If the reaction norm is a set of phenotypes optimized for each environmental state, its evolution is not significant from a theoretical viewpoint. In such a case, the reaction norm is an assembly of optimal solutions for the given environmental state which can be understood by a simple optimization approach.

However, evolution of the reaction norm is probably restricted by some factors. One possible constraint is a pleitoropy. When an organism changes its phenotype for one environmental state it may necessarily influence phenotypes for other states to some degree due to restrictions in gene expression or developmental processes. Another possible constraint is the cost of achieving the plasticity of phenotype. The organism may have to invest some cost to achieve mechanisms for plasticity, to develop the sensory system for environmental cue and the formation process of alternative phenotypes. Accordingly, an analysis of reaction norm evolution requires a different approach from simple optimization. Indeed, reaction norm evolution has been theoretically analyzed by some approaches that considered those constraints, *i.e.*, optimization, quantitative genetic model and gametic model (Scheiner, 1993).

I analyzed the optimal reaction norm from the viewpoint of optimization. In the model, an individual phenotype is divided into two components as $v_0+u(x)$. In this formulation, v_0 is a basal phenotypic level independent of environmental state, whereas u(x) is an additional response to a given discrete environmental state x. Under the given environment state x, the phenotype $v_0+u(x)$ is considered to result in a success f(x),

$$v_0+u(x)$$
), accompanied by the total cost $c=k_1v_0+k_2\sum_{x=1}^Xu(x)^2$. The optimal reaction norm is obtained by

maximizing $\Phi = f(x, v_0 + u(x)) - c$ with respect to v_0 and u(x). Based on this approach, it is possible to discuss the general properties of the optimal reaction norm. Phenotypic plasticity may be intuitively expected to be advantageous under a significantly variable environment. However, I found that phenotypic plasticity may become less likely when environmental variations is large.