What is The Best Path: Lesson Plan (7th & 8th Grade)

Objective:
- Use mathematical modeling to build the best delivery path

Materials:
- Which Path to Take? Activity Sheet

Instructional Plan:
The Letter Carrier problem draws on students' prior experiences:
A letter carrier needs to deliver mail to both sides of a street. What path should the carrier choose?

The students should decide if the distance of the total path is key in deciding the best path. Discuss the possible locations of the mailboxes on the lots of houses. Discuss possible assumptions.

Decide how the sizes of the width and length of the street as well as the number of mailboxes on one side affect possible solutions. These are denoted by ***w, n*** and ***l*** in the diagram.

One may decide to take one of ***w, n*** and ***l*** to be constant to make this problem more accessible. Or, ask the students to assign values to ***w, n*** and ***l***, then work the problem. For example, try setting ***n*** =10.

Tennessee Mathematics Standards:

8th grade

Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems.

Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions.

Students solve real-world and mathematical problems leading to two linear equations in two variables.

7th grade

Students use variables to represent quantities and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions.

Adapted from Czocher, J., and D. Moss. 2017. Mathematical modeling: Are prior experiences important? Mathematics Teacher 110: 654-660.

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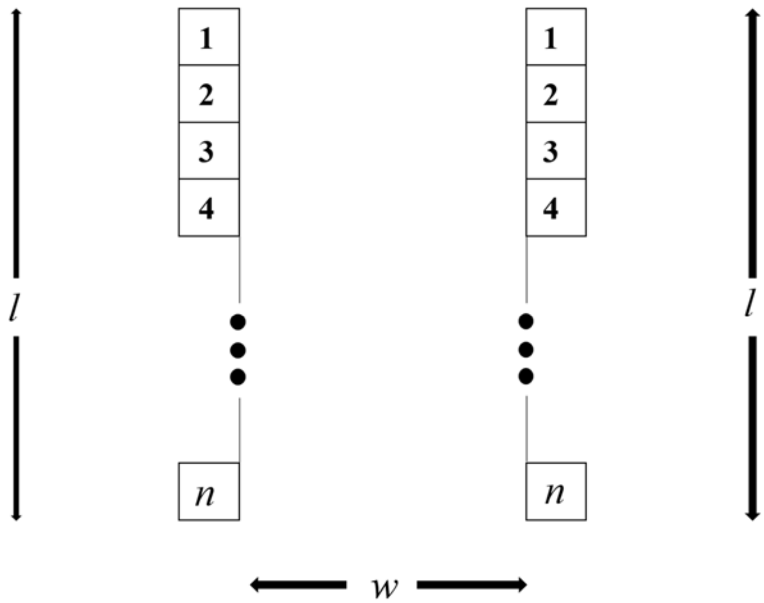
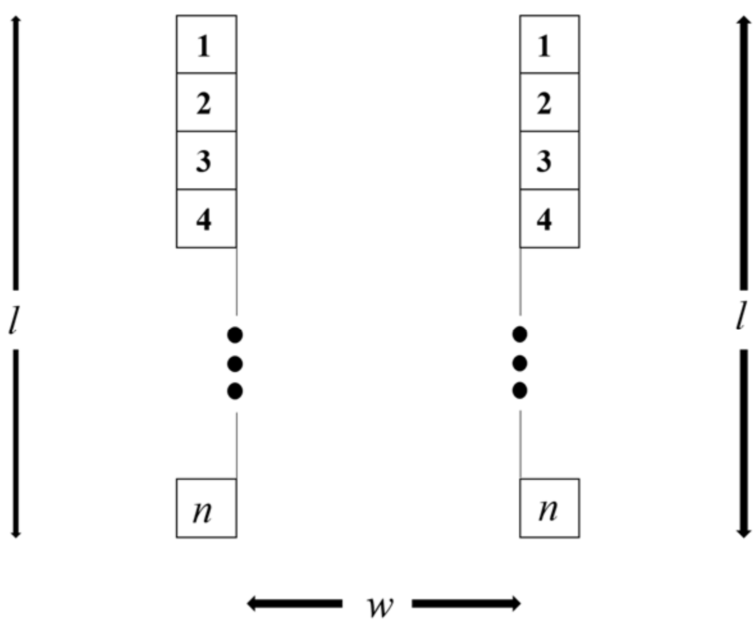
Which path to take? Activity sheet

A mail carrier needs to deliver mail to both sides of the street. What path should the carrier take? List your assumptions. Calculate at least two different paths to help with choosing your solution and explain why you chose the path you did. Ask yourself, when is this path the best path?

1st Path:



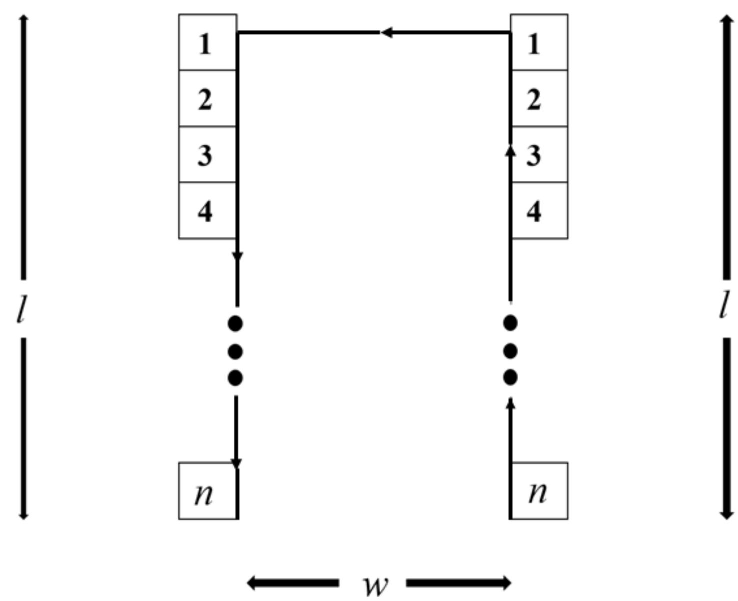
2nd Path



Possible solutions, assuming the mailboxes are in the middle of each lot.

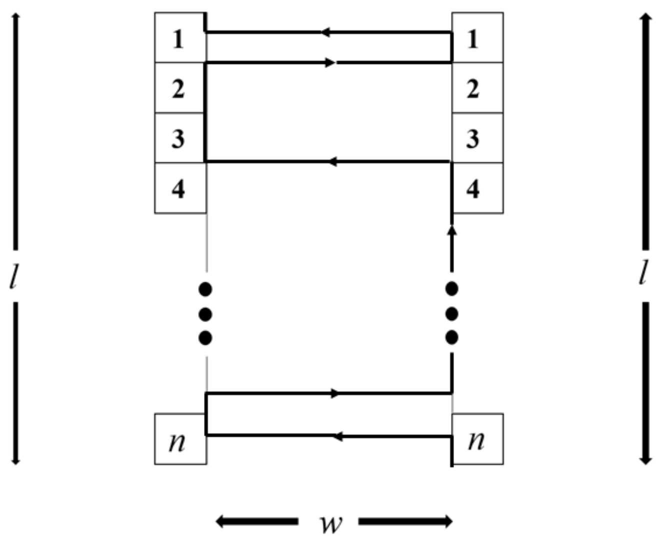
1. Start on one side of the street and go to all mailboxes on that side. Then cross the street and go down the other side to go to the remaining mailboxes.

Distance of path $D_1 = 2l + w - l/n$ Note that the length of the lots are l/n.



2. Start at the bottom of the street on one side. Go one mailbox and then cross the street. Next go to 2 mailboxes on that side and then cross the street. Then go to two mailboxes on that side and afterwards cross the street. Continue until you are at the end of the street on the opposite side where you started.

Distance of path $D_2 = nw + l$. Note that the street of width w is crossed n times. The length of the street is added.



When is the first path shorter? $2l + w - l/n < nw + l$

$$l - l/n < nw - w$$

$$(l/n)(n-1) < w(n-1) \qquad \text{when } l/n < w.$$

Note that algebra is easier if $n = 10$.

Then $2l - w - l/10 < 10w + l$

$$l - l/10 < 9w$$

$$(9/10)\, l < 9w$$

$$l/10 < w$$