Trout Modeling Example

Goals:

1. Engage students in mathematical modeling with an open-ended situation.
2. Use of sequence notation for linear difference equations. Use recursive models. Use of EXCEL as needed.
3. Extension: The concept of equilbrium value. What limit does a sequence approach (if any)?

TASK

Lenhart Lake is stocked with trout. As time goes on, how will the trout population change?

1. What questions would you or your students ask to start on such a problem?

   Formulate a scenario and the correspondg question or problem.

2. Define the question more clearly. Make assumptions.

3. Define the variables. Make a model.

4. Solve the question that you formulated. Use Excel if needed.

5. What could you have done differently? How would you report your solution? Use Excel if needed.

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References

A. M. Alhammouri, G D. Foley and K. Dael, Tracking Trout: Engaging Students in Modeling, NCTM Mathematics Teacher 2018, April, 416-423.

E. N. Bodine, S. Lenhart, and L. J. Gross, Mathematics for Life Sciences, Princeton University Press, 2014.

Problems to work in teams

1. A herd of newly introduced elk in a wildlife management area in the Cumberland Mountains in Tennessee increases its numbers by 10% each year. Let x_n be the number in the population at the end of year n.
   (a) Find the difference equation relating x_{n+1} to x_n.
   (b) Solve for x_n if $x_0 = 50$.
2. The probability that an adult elk survives the year is 90%. Suppose that a herd starts with 50 adult individuals, and let x_n be the number of original adult elk still alive after n years. Assume no individuals are added to the population.
   (a) Write the difference equation relating x_{n+1} to x_n.
   (b) Find the general solution to the difference equation.
   (c) Find the number of original adult elk in the herd after 4 years.
3. The body eliminates 10% of the amount of pain reliever drug present each hour. Let x_n be the amount of drug (in milligrams) in the body n hours after the initial dose of 180 mg.
   (a) Relate x_{n+1} to x_n.
   (b) Find the general solution to the difference equation.
   (c) How many milligrams of the drug remains in the body after 1 hours?
4. A population of buffalo can increase its numbers by about 10% each year. Let x_n be the population count after n years, and assume that h buffalo are removed from the herd at the end of each year.
   (a) Find x_n if $x_0 = 1000$ if $h = 20$.
   (b) With $h = 20$, would this population ever go negative? (If so, the model would not make biological sense.)
   (c) Again assuming $x_0 = 1000$, find the largest h so that $x_{10} \geq 1500$.
5. Suppose in a lake, a population of trout increases its own numbers by 10% each year. After the births occur each year, to build up the population, 100 young trout are added each year. Let x_n denote the size of the population after n years, starting with a population of 1000 trout.
   (a) When does $x_n \geq 2000$ happen?
   (b) After that time, the lake is no longer stocked and fishermen will catch 400 fish per year (not worrying about the size of the fish). What is the fate of the population?

ANSWERS

1. **(a)** $x_{n+1} = 1.1 \cdot x_n$ **(b)** $x_n = 1.1^n \cdot 50$
2. **(a)** $x_{n+1} = 0.9 \cdot x_n$ **(b)** $x_n = 0.9^n \cdot 50$ **(c)** $x_4 \approx 33$
3. **(a)** $x_{n+1} = 0.9 \cdot x_n$ **(b)** $x_n = 0.9^n \cdot 180$ **(c)** 162 mg
4. **(a)** $x_n = (800)1.1^n + 200$ **(b)** No **(c)** Use $x_n = (1000 - 10h)(1.1)^{10} + 10h \geq 1500$ and obtain $h = 68$
5. **(a)** $x_{n+1} = 1.1x_n + 100$ and $x_n = (1000 - \frac{100}{1-1.1})(1.1)^n + \frac{100}{1-1.1}$. Let $x_n = 2000$ and solve for $n = 5$ (round up). **(b)** Start with $x_0 = 2000$ and use $x_{n+1} = 1.1x_n - 400$ and see that the stock drops to 0. extinction