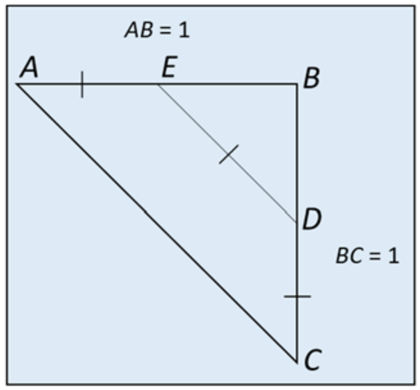
Using Algebra to Solve a Geometry Problem with Two Strategies

Consider the isosceles right triangle *ABC* where *AB* = *BC* = 1 (Figure 1). Find *CD* so that *AE* = *DE* = *CD*.

Figure 1. Isosceles triangle *ABC*.



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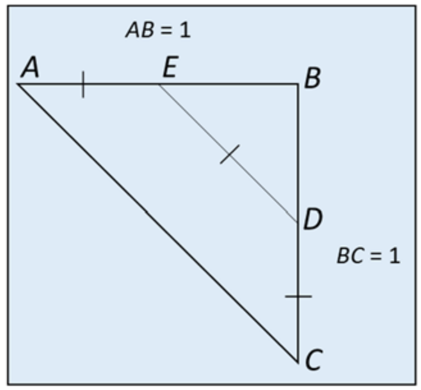

Strategy 1

Let *BD* = *BE* = *x*. Since triangle *EBD* is also an isosceles right triangle, using $x^2 + x^2 = (DE)^2$, then $DE = x\sqrt{2}$. Also, *CD* the is 1 – *x*. If *CD* = *DE*, then $1 - x = x\sqrt{2}$. Solving for *x* leads to

$$x = \frac{1}{1 + \sqrt{2}} = \sqrt{2} - 1 = BD.$$

Since *CD* = 1 – *x*, $CD = 2 - \sqrt{2}$. Additional confirmation that this solution is correct comes from the observation that

$$DE = x\sqrt{2} = \sqrt{2}(\sqrt{2} - 1) = 2 - \sqrt{2}.$$

Strategy 2

Another solution could be to label *BD* = *y*. Then, use the Pythagorean theorem to find the value of *y* in the isosceles right triangle *EBD*. *BD* = *BE* = 1 – *y* leads to

$$(1 - y)^2 + (1 - y)^2 = y^2 \text{ or}$$
$$2y^2 - 4y + 2 = y^2 \text{ or}$$
$$y^2 - 4y + 2 = 0.$$

Working with the quadratic formula results in two solutions

$$2 - \sqrt{2} \text{ and } 2 + \sqrt{2}.$$

Since $2 + \sqrt{2} > 1$, this is rejected as a solution. $CD = 2 - \sqrt{2}$.

Adapted by Greg Wiggins and Suzanne Lenhart from Marion. 2018. Two Problem-Solving Strategies Pay Off. Mathematics Teacher 111: 549-555.