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Biology Meets Math

Predator-Prey Relationships in Belowground Ecosystems

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US Department of Homeland Security

USDA

Goals:

- Define Predator and Prey in relation to soil ecology
- Define a mathematical model and identify some examples when one is useful
- Create a hypothesis
- Explain the basics behind the given simple Predator-Prey Relationship Model
- Graph the results of the given model
- Use your results to support or refute your hypothesis

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Predators & Prey

Predator: an organism that hunts, kills and eats other organisms (prey) to survive

Prey: an organism hunted and taken as food

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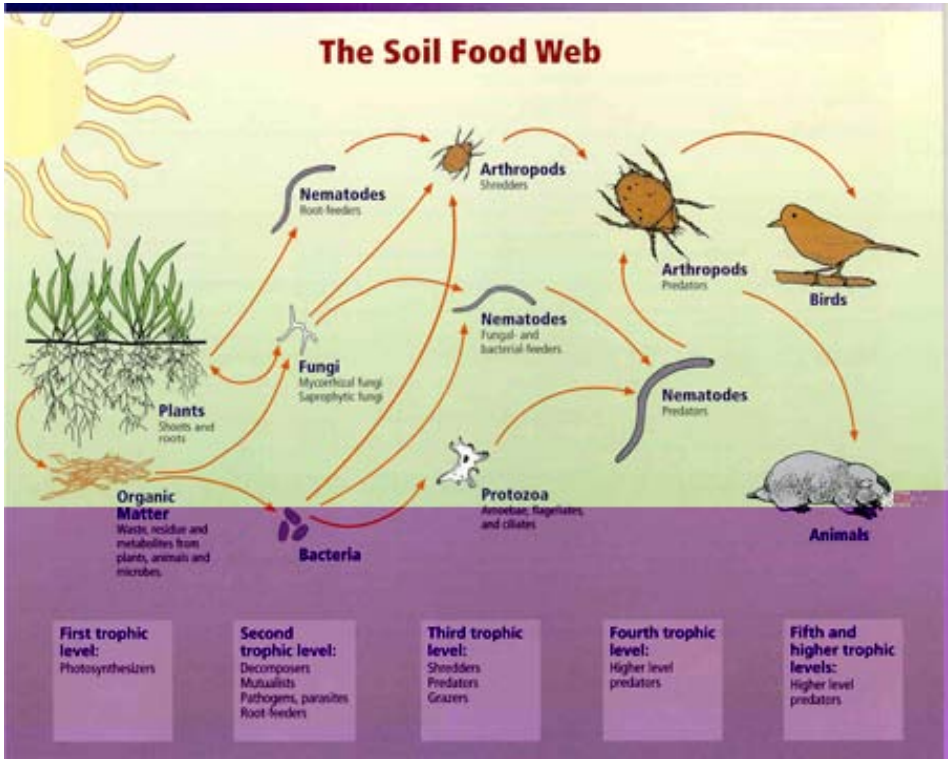
In the Soil ….



Source: TIEE (ESA)

Predation happens on a variety of scales

"Where the telescope ends, the microscope begins. Which of the two has the grander view?" -- Victor Hugo

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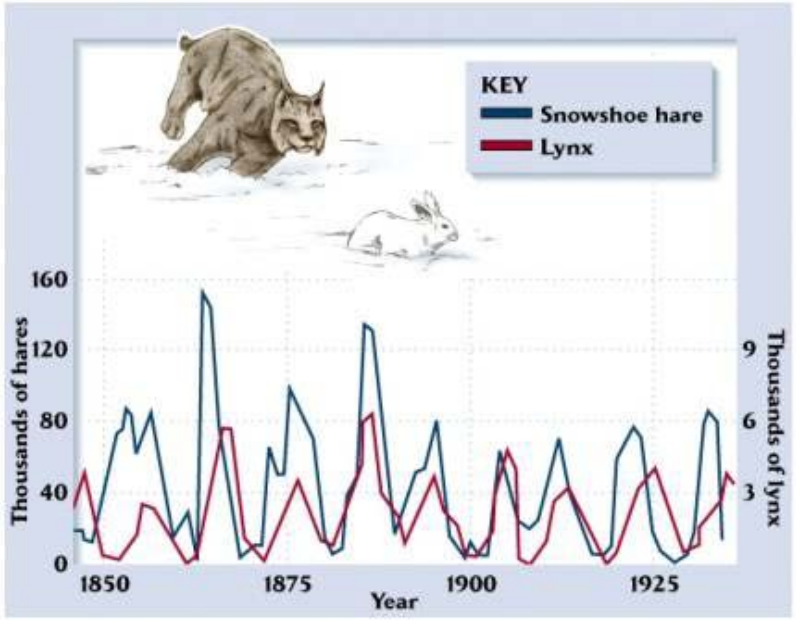
Classic Predator-Prey



- Canadian lynxes and hares
- Fur trapping data
- Controversial!
- Theoretically you'd expect prey to peak before predators every time
- Let's get help from a model!

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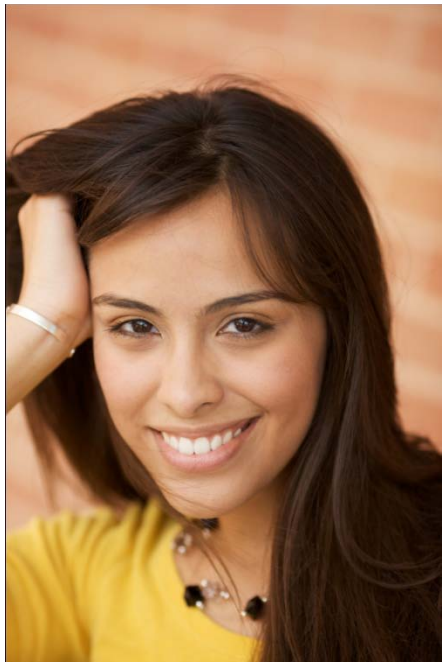
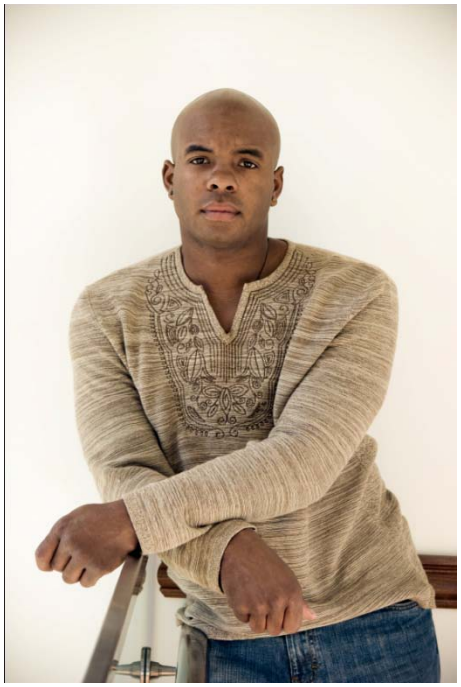
Do these models look helpful?





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So what do we mean by a Model?

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You're a Modeler!

- Have you ever calculated how much gas it's going to take you to get somewhere?
- Have you ever estimated how long it'll take you to save up for something?
- Picked the best line at the grocery store?
- Others?

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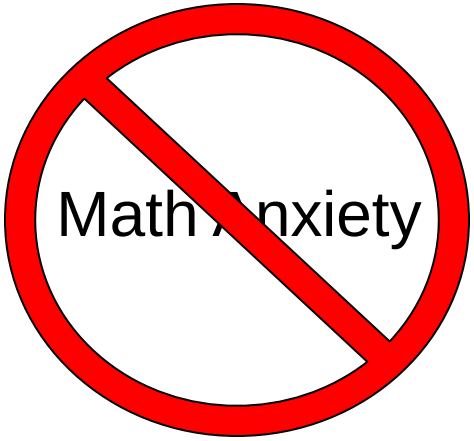
Brace Yourself for Math!

(Trust Me, You Can Do It!)



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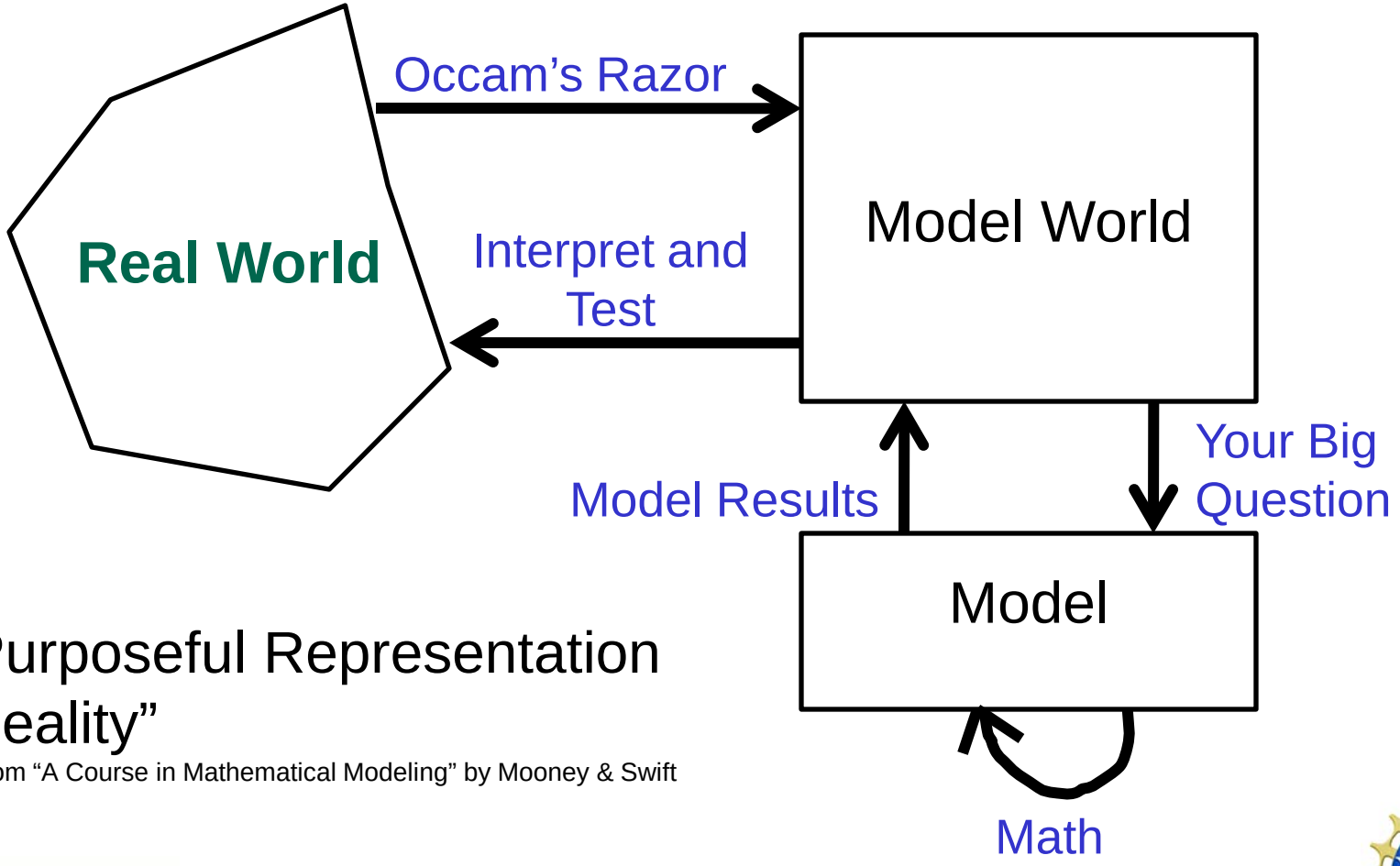
What do we mean by a Mathematical Model?



"A Purposeful Representation of Reality"

Figure from "A Course in Mathematical Modeling" by Mooney & Swift

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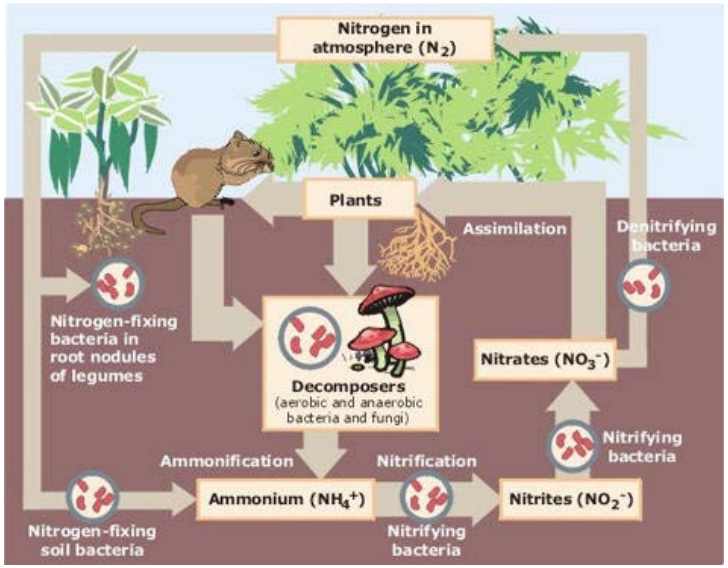
Soil Biologists Use Models

- Nutrient cycling
- Decomposition
- Carbon sequestration
- Predator-Prey
- Host-parasite
- Soil formation/erosion



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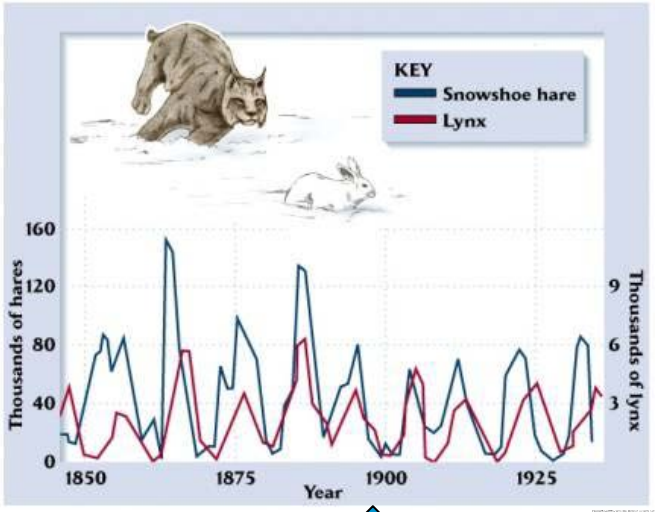
Some Kinds of Models

- Stochastic Model: Has randomness!
- Discrete Model: No randomness
- Theoretical Model: for explaining observed phenomena
- Deterministic Model: for predicting events in time

Theoretical Predator-Prey



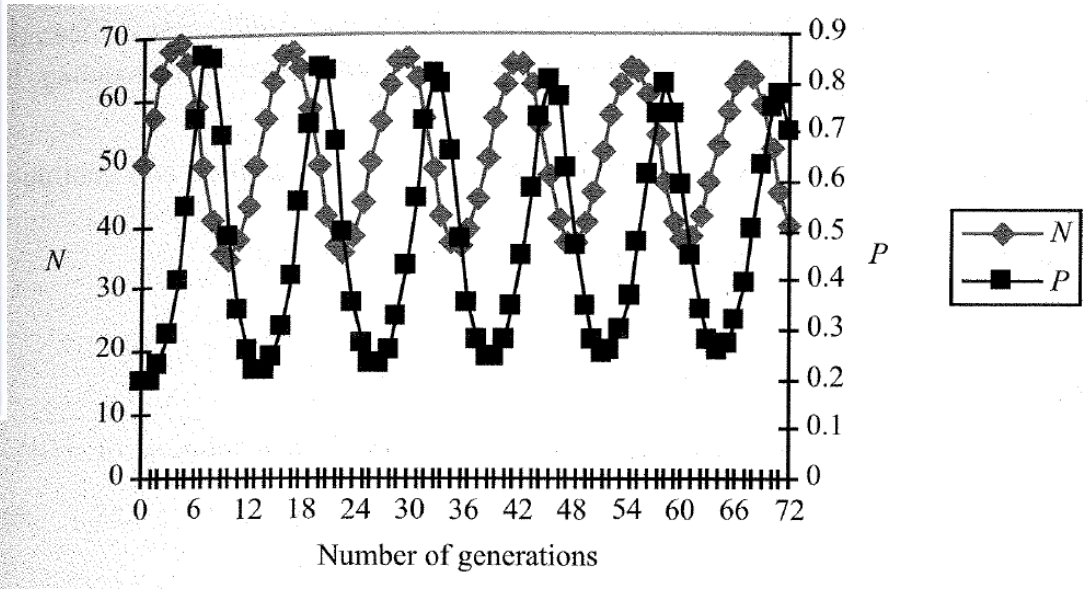


FIGURE 1.23 Simulation of a Discrete Predator-prey Model with $K = 100$, $R = 1.5$, $Q = 0.02$, $N(0) = 50$, and $P(0) = 0.2$.

How did they do it?

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The Mathematical Model

Terms:

N_n Your prey population at the moment (time step n)

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N_{n+1} Your prey population at the next time step

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R The prey population's growth rate

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R The prey population's growth rate

K The prey's carrying capacity

The Mathematical Model

Terms:

N_n Your prey population at the moment (time step n)

N_{n+1} Your prey population at the next time step

R The prey population's growth rate

K The prey's carrying capacity

C The predator's efficiency in nabbing prey

The Mathematical Model

Terms:

N_n Your prey population at the moment (time step n)

N_{n+1} Your prey population at the next time step

R The prey population's growth rate

K The prey's carrying capacity

C The predator's efficiency in nabbing prey

P_n Your predator population

The Mathematical Model

Terms:

N_n Your prey population at the moment (time step n)

N_{n+1} Your prey population at the next time step

R The prey population's growth rate

K The prey's carrying capacity

C The predator's efficiency in nabbing prey

P_n Your predator population

Q The predator's efficiency in using prey to reproduce

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How Do You Expect These to Relate?

- If the prey population growth rate is positive, what do you expect will happen to the population of prey over time?
- As the prey population reaches its carrying capacity, what do you expect will happen to the prey population?
- As the predator's efficiency in getting prey goes up, what do you expect will happen to the prey population?
- As the predator's population goes up, what do you expect will happen to the prey population?
- As the prey population goes up, what do you expect will happen to the predator population?
- As the predator's efficiency in using energy it gets from prey to reproduce goes up, what would happen to the prey population?

How Do You Expect These to Relate?

- If the prey population growth rate is positive, what do you expect will happen to the population of prey over time? Prey population goes up.
- As the prey population reaches its carrying capacity, what do you expect will happen to the prey population? Prey growth slows down.
- As the predator's efficiency in getting prey goes up, what do you expect will happen to the prey population? Prey population goes down.
- As the predator's population goes up, what do you expect will happen to the prey population? Prey population slows down or goes down.
- As the prey population goes up, what do you expect will happen to the predator population? Predator population goes up.
- As the predator's efficiency in using energy it gets from prey to reproduce goes up, what would happen to the predator population? Predator population goes up.

Introducing … what you just said

$$N_{n+1} = R\left(1 - \frac{N_n}{K}\right)N_n + N_n - CN_nP_n$$

Growth of Prey Population

Existing Prey Population

Prey Population Gets Pulled Down by Predators

What about the predator population?

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Hello Predators!

$$N_{n+1} = R\left(1 - \frac{N_n}{K}\right)N_n + N_n - CN_nP_n$$

$$P_{n+1} = \underbrace{QN_nP_n}$$

Growth of Predator Population

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Does this actually work?

$$N_{n+1} = R\left(1 - \frac{N_n}{K}\right)N_n + N_n - CN_nP_n$$

$$P_{n+1} = QN_nP_n$$

Let's plug some stuff in!

Assumptions:
K =100
R = 1.5
Q = 0.02
N_0= 50
P = 0.2
C = 3

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Answer Key

Time Step (n)	N (Prey Population)	P (Predator Density)
0	50.0	0.20
1	57.5	0.20
2	59.7	0.23
3	54.6	0.27
4	47.6	0.29
5	43.6	0.28
6	43.9	0.24
7	49.2	0.21
8	55.7	0.21
9	57.6	0.23
10	54.5	0.26

Goals:

- Play with a Predator-Prey Model using Netlogo (free online software!)
- Have some free-wheeling inquiry-based fun

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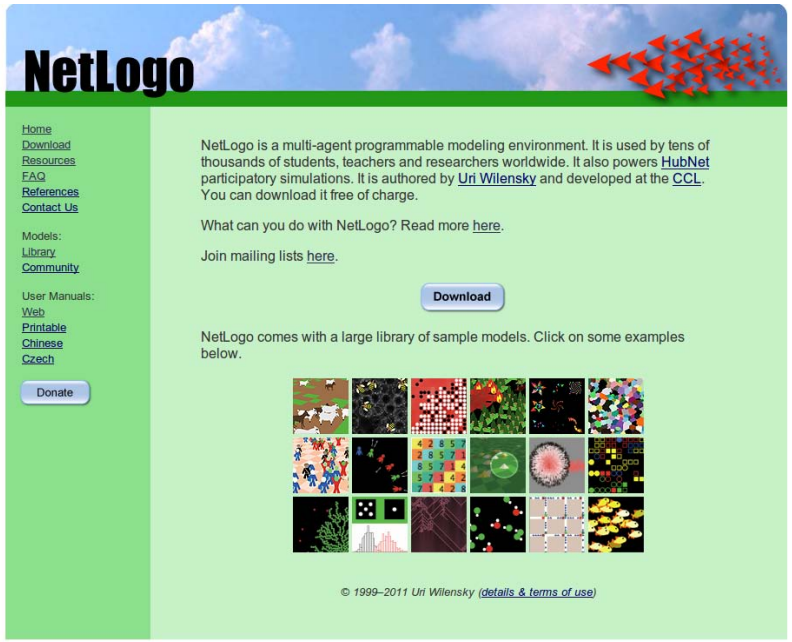
Computers: An Easier Way to Model



- Go to website:

ccl.northwestern.edu/netlogo/

- Download the program

- Pay attention to what folder the program downloads into. Then put the file

Bacteria Protozoa Predation.nlogo

in the same folder

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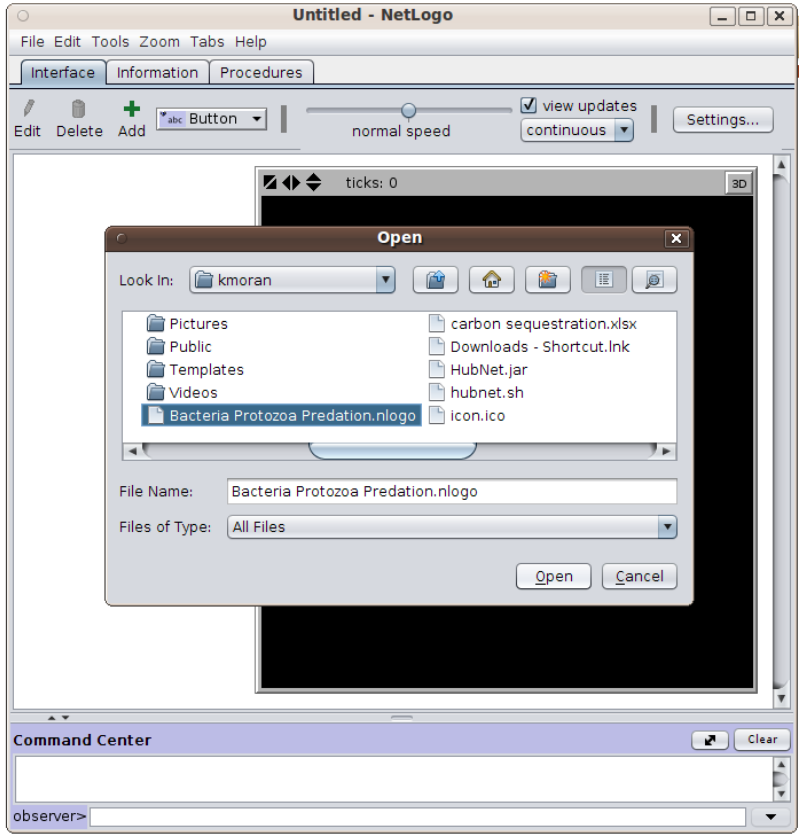
Open the Model



- Run Netlogo
- Go to File → Open
- Find the Bacteria Protozoa file
- Select Open

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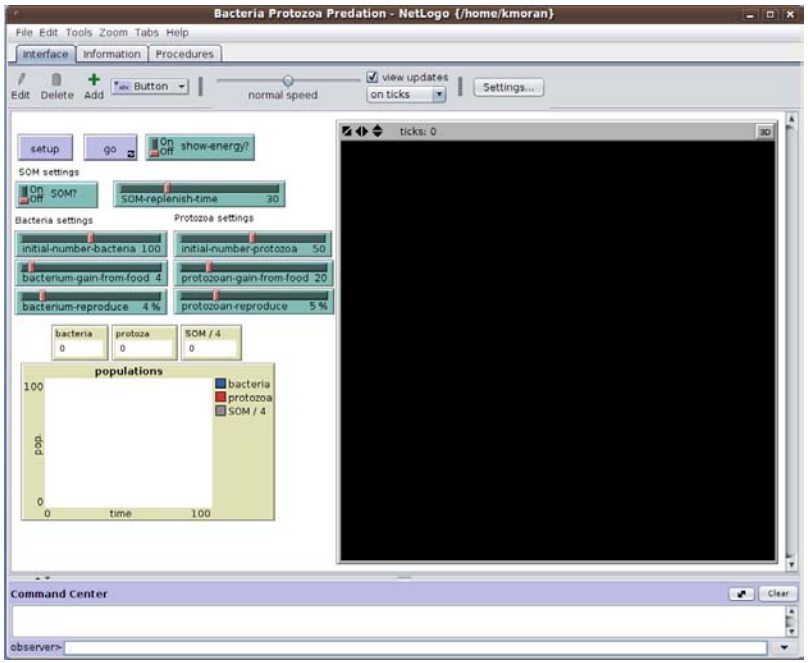
Netlogo Interface



- Check it out!
- To run …

Press the Setup button

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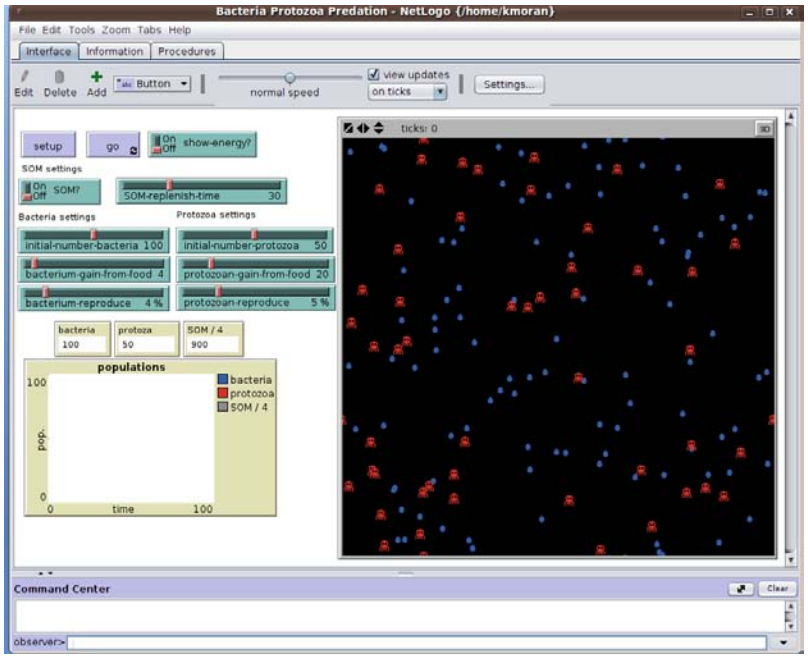
Netlogo Interface



- Check it out!
- To run …

Press the Setup button

Press the Go button

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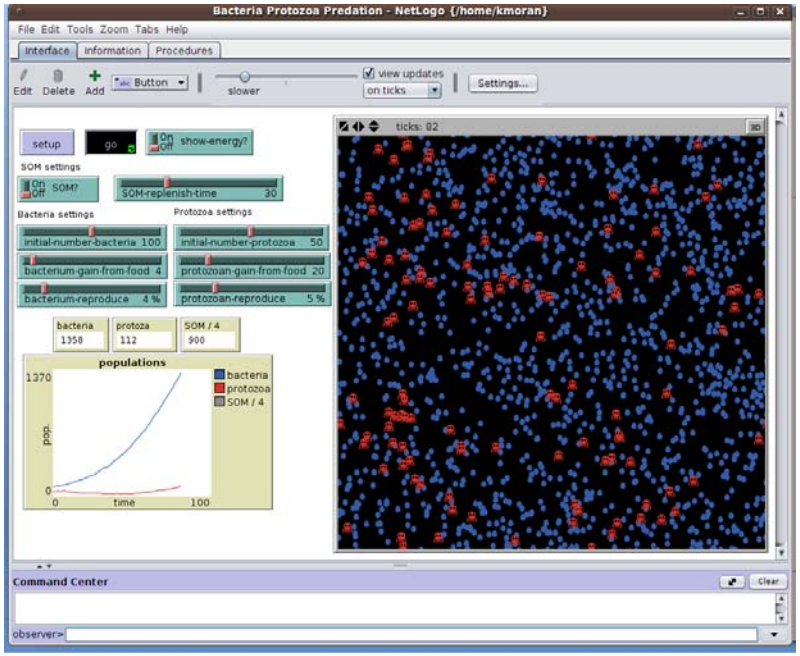
Netlogo Interface



- Check it out!
- To run …

Press the Setup button

Press the Go button

And watch what happens!

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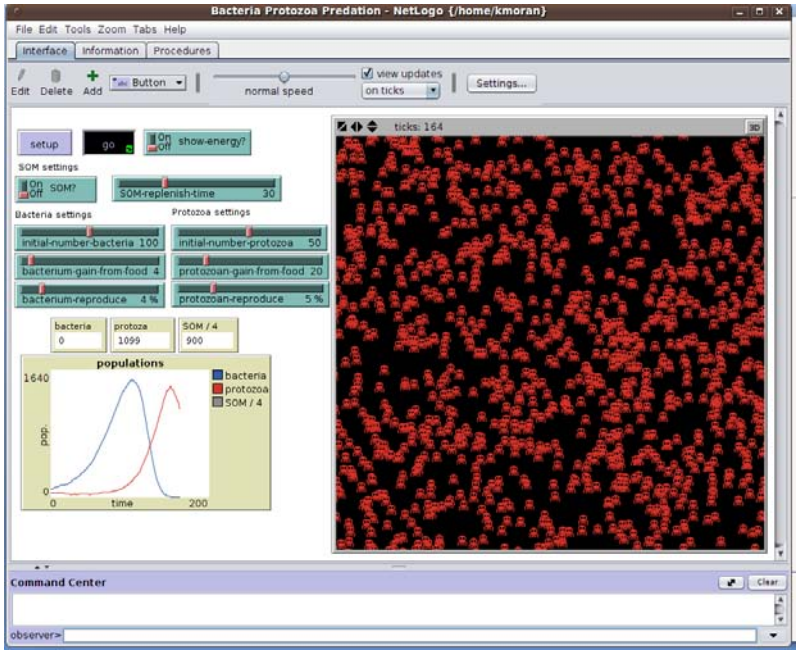
Netlogo Interface



- Check it out!
- To run …

Press the Setup button

Press the Go button

And watch what happens!

Oh my.

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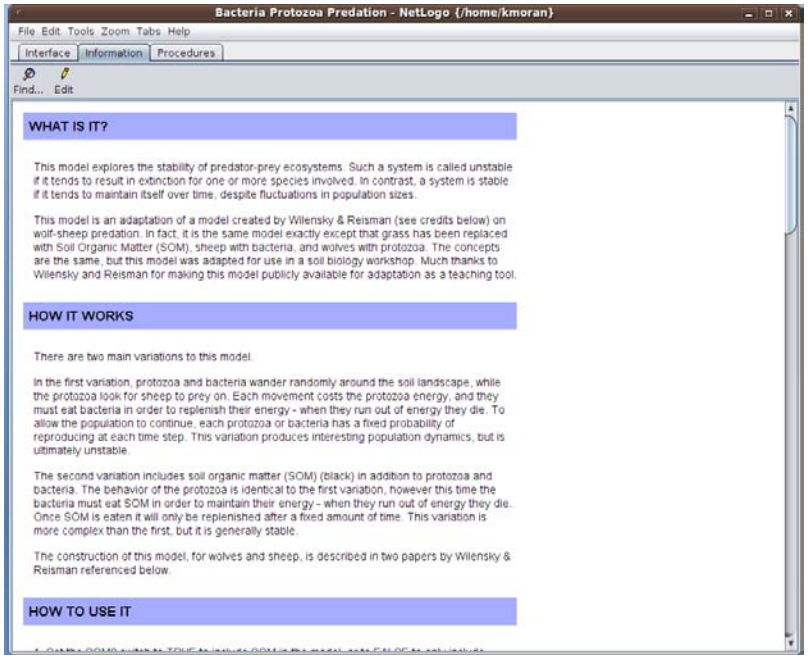
Background on the Model



- Select the Information tab, and read about this model
- Then work through the worksheet

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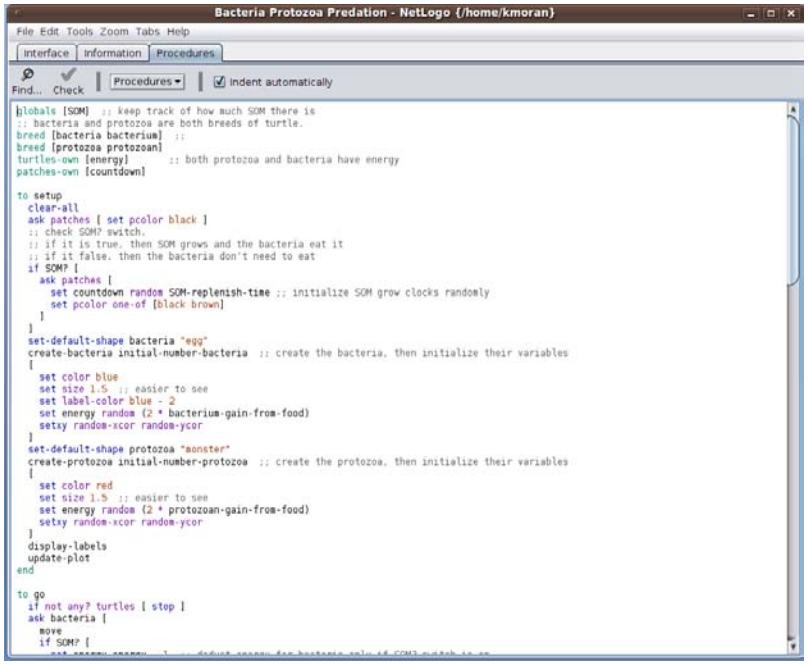
Need More Challenge?



- Click on the Procedures tab
- See the computer codes that make the model work
- Tutorial available on the Netlogo website

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Looking for More?

- Check out our modules on quantifying biodiversity and measuring a forest!

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Sources

Charles J. Krebs. *Ecology: The Experimental Analysis of Distribution and Abundance*. Harper and Row Publishers, New York, second edition, 1978.

Douglas Mooney & Randall Swift. *A Course in Mathematical Modeling*. The Mathematical Association of America, 1999.

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For this Module & More:

- Website: www.nimbios.org
- See what we're all about
- Sign up for our bimonthly email newsletter
- Check our blog

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