# Final Report Submitted by NIMBioS Sabbatical Visitor: Vlastimil Krivan

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#### Research:

- 1) Revision of Krivan (2010). In this article the patch and diet choice models of the optimal foraging theory are reanalyzed with respect to evolutionary stability of the optimal foraging strategies. In their original setting these fundamental models consider a single consumer only and the resulting fitness functions are both frequency and density independent. Such fitness functions do not allow us to apply the classical game theoretical methods to study an evolutionary stability of optimal foraging strategies for competing animals. In this article frequency and density dependent fitness functions of optimal foraging are derived by separation of time scales in an underlying population dynamical model and corresponding evolutionarily stable strategies are calculated. Contrary to the classical foraging models the results of the present article predict that partial preferences occur in optimal foraging strategies as a consequence of the ecological feedback of consumer preferences on consumer fitness. In the case of the patch occupation model these partial preferences correspond to the ideal free distribution concept while in the case of the diet choice model they correspond to the partial inclusion of the less profitable prey type in predators diet.
- 2) Preparation of article (Krivan, submitted) on the Gause model. This article re-analyses a prey-predator model with a refuge introduced by one of the founders of population ecology G. F. Gause and his co-workers to explain discrepancies between their observations and predictions of the Lotka-Volterra prey-predator model. They replaced the predator linear functional response used by Lotka and Volterra by a saturating functional response with a discontinuity at a critical prey density. At concentrations below this critical density prey were effectively in a refuge while at higher densities they were available to predators. Thus, their functional response is of the Holling type III. They analysed this model and predicted existence of a limit cycle in predator-prey dynamics. However, this article shows that their model is ill posed, because trajectories are not well defined. Using the Filippov method, solutions are defined and analysed. The analysis shows that depending on parameter values, there are three possibilities: (1) trajectories converge to a limit cycle, as predicted by F. G. Gause, (2) trajectories converge to an equilibrium, or (3) the prey population escapes predator control and grows to infinity.
- 3) I have been working on a text on dynamic models of evolutionary game theory

## New collaboration:

- 1) I started collaboration with prof. Joel Brown, whom I met during one workshop at NIMBioS, on a model describing animal distribution under vigilance.
- 2) I started collaboration with the Forest Insect working group. In particular, I have been developing two new models pertinent to the subject of this working group.
- 3) I met prof. Robert Holt and we discussed several possible ways how to collaborate in future. We also exchanged some manuscripts which can possibly lead to more joint work in future.

## Continuing collaboration:

1) I met with prof. Ross Cressman and we started a new research project related to modelling evolution of dispersal using multi-valued maps. This project will continue and it is likely to new results.

## Talks:

- 1. "On the Lotka-Volterra foraging games ", National Institute for Mathematical and Biological Synthesis, University of Tennessee, Knoxville, August 31, 2010
- 2. "On The struggle for existence", Mathematical Biology Seminar, University of Tennessee, Knoxville, September 21, 2010
- 3. "On the Lotka-Volterra foraging games ", University of Ilinois, Chicago, November 22, 2010

## References

Krivan, V. 2010. Evolutionary stability of optimal foraging: partial preferences in the diet and patch models. Journal of theoretical Biology 267:486-494.

Krivan, V. (subm). On the Gause predator-prey model with a refuge: A fresh look at the history.