MODELING THE FOREST

by Kelly Sturner, Barry Golden, and Suzanne Lenhart

"This tree is five inches thick!" says Amy, age 12. Next to her, Jason announces, "I've found the biggest one here—10 inches!" Another student carefully logs these data into her science notebook and says, "Based on these measurements, I think this stand of trees is going to have bigger trunks than the one on the other side of the school yard!"

Measuring and modeling trees is an engaging way for students to make the connection that mathematics is the language of science. It is also an opportunity to bridge related concepts recently emphasized in *A Framework for K–12 Science Education: Practices, Crosscutting Concepts, and Core Ideas* (NRC 2012), as well as the *Common Core State Standards, Mathematics* (NGAC and CCSSO 2010) (Figure 1). When students design their own authentic investigation of a stand of trees or forest that is nearby and significant to them, they inherently apply some of the *Framework*'s scientific practices, such as "developing and using models" and "us-

FIGURE 1	Relating measuring trees to standards

This activity makes connections between developing and using models and using mathematical and computational thinking, but there are many related standards that could be explored:

Next Generation Science Standards (Achieve Inc. 2013)

- Dimension 1: Scientific and engineering practices (all practices)
- Dimension 2: Crosscutting concepts (1. Patterns; 3. Scale, proportion and quantity; 4. Systems and system models; 6. Structure and function)
- Dimension 3: Core ideas in life sciences (LS.1: From molecules to organisms: Structures and processes; LS.2: Ecosystems: Interactions, energy, and dynamics)

Common Core State Standards, Mathematics (NGAC and CCSSO 2010)

- 6.RP. Understand ratio concepts and use ratio reasoning to solve problems.
- 6.G. Solve real-world and mathematical problems involving area, surface area, and volume.
- 6.SP. Summarize and describe distributions.
- 7.RP. Analyze proportional relationships and use them to solve real-world problems.
- 7.SP. Draw informal comparative inferences about two populations.

ing mathematics and computational thinking" (NRC 2012). Environmental education that focuses on local natural areas that are more relevant to students can lead to the development of environmental sensitivity (Sward and Marcinkowski 2005). In this article, we present a lesson using mathematical modeling, as well as a rationale for its use in forest science. Just like forest scientists, students can collect tree data to model a single tree or a forested area. Students can apply these measurements and models to monitor and compare stands of trees in their school yard, their backyard, a park, or elsewhere.

This lesson would fit well within middle school curricula on organisms (growth, structure, and processes), as it explores gathering data on the growth of trees and how this requires understanding their structure and growth patterns. It would also fit within a unit on ecosystems (interactions, energy, and dynamics) by emphasizing how developing a model can help in understanding the response of trees to resource availability or other environmental factors. Students will be most successful completing this activity if they understand how to take precise measurements, how to use significant figures, and how to use appropriate units. Reviewing the concept of ratios and their relevance to science would also be appropriate (for example, the length of one side of a forest area may be m, the area of the forest may be m^2, and the area of forest cleared in a day by deforestation could be m^2/day).

Starting the lesson with a single tree

A great way to begin the lesson is to discuss with students reasons why one would want to measure or monitor trees in a forest (see Sidebar, p. 6), choosing one or two reasons that are especially relevant to your region. As a hook for more advanced students, you might show and discuss a video of an ecologist talking about research on trees (see Resources).

To find out what students may know already, we suggest facilitating a discussion regarding what kinds of data could be collected about individual trees. You might show students pictures of trees and allow them, working in groups, to observe how trees, even trees of the same age or species, come in all shapes and sizes (for suggested sources of tree images, see Resources). Use Part A of the Activity Worksheet to guide the groups in their preliminary discussions. After 10 minutes of group discussion, bring the class back together and have students share their group's ideas, listing unique points on the board. During the class discussion, you could also ask students which of their measurements would be fairly easy to make, which ones would be difficult, and why. Suggest a way to make the difficult measurements using simple tools and math (measurement tapes, data tables, and the clinometer for height, discussed below).

In addition, with the introduction of each new idea about how trees can vary, ask students to explain some reasons why each factor might vary (e.g., tree age, resource availability, impact from humans or disease, crowding from other trees).

Encourage students to propose and argue for their ideas on how to make these measurements and decide on methods together. Students should consider what biases might be introduced in the methods they select and describe the methods precisely: How will they hold the measuring tape? Is it OK if it droops in the middle, or if different people measure different trees? What units will they use?

Students may realize quickly that one method is to break trees down into their parts—trunk and crown (all of the branches). They can think of the trunk as being like a cylinder and the crown like a big ellipsoid on top (often not spherical) (Figure 2).

When students discuss measuring tree heights, you might point out that it is both unsafe and time consuming to climb trees to measure their heights. Ask students to come up with ideas for another way to do this. Foresters have special ways for measuring the heights of tall trees with a clinometer, a tool that uses principles of trigonometry. Trigonometry is a high school concept in the *Common Core State Standards, Mathematics* (NGAC and CCSSO 2010), though some advanced middle school students may have been introduced to the terminology. If students are not yet familiar with trigonometry, it is fine to not delve too deeply into how the clinometer works; instead, simply offer it as a tool that takes advantage of principles of mathematics they will learn someday. (For more information, see How to Use a Clinometer in Resources.)

Students may enjoy making their own clinometers using a protractor, straw, string, and weight (see Schoolyard Clinometer in Resources), or clinometers are available for purchase from science supply companies. If your class has access to such technology, there are clinometer apps available for iPad and iPhone.

A clinometer may also be used to measure the size of the crown, another place students may have observed that trees can vary. Astute students can figure out that this tool may also be used to measure the height of the trunk from the base to the crown, which can then be used to calculate the height of the crown by difference (Figure 3).

Less obvious to students may be the possible measurements of tree width. Asking students if a tree is a three-dimensional object and what that means, or asking them to define the difference between height and width, can start them down this path of reasoning. Common tree-width measurements taken by forest scientists are shown in Figure 3. To get them thinking about why crowns may vary, show students pictures of trees growing in dense forests compared to those growing in open woodland savannahs. They will observe that trees in open areas

FIGURE 2 Trees of all shapes

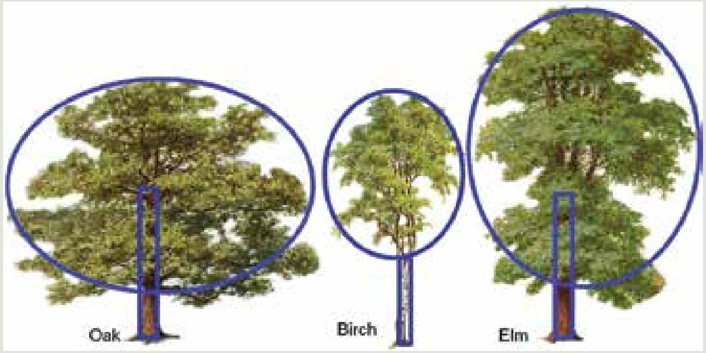
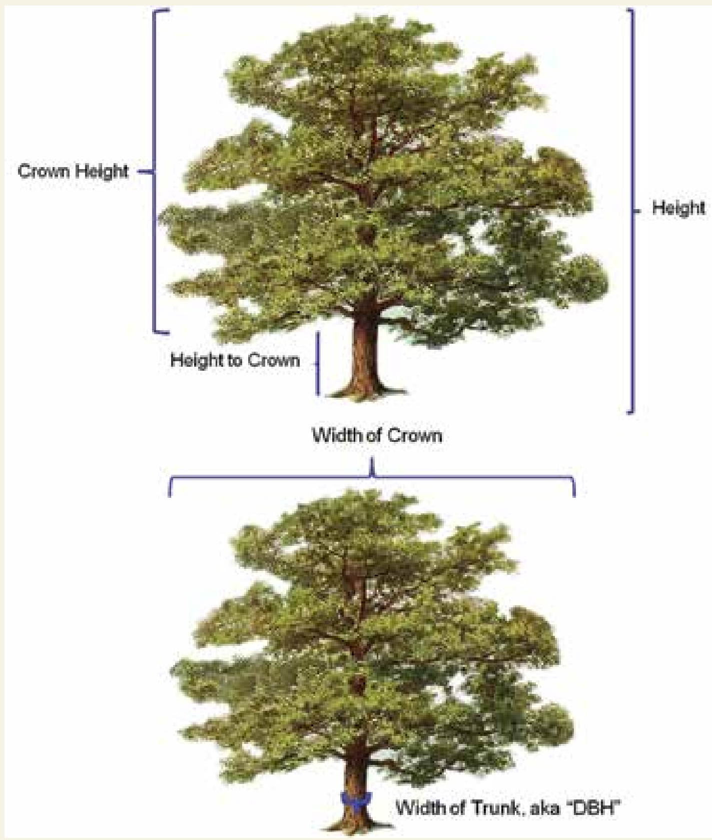


FIGURE 3 Tree height and width measurements



Why do scientists model and monitor forests?

Many different kinds of forest scientists (see below) collect data that help them to monitor the growth and development of forests and human impacts on this important ecosystem.

One reason is to determine a forest's economic value—forests are important natural resources that provide many products. The United States produces over $200 billion worth of wood and paper products annually (SLMA). Responsible foresters need to make sure we manage this resource sustainably so that it continues to provide materials for future generations. Forests are also important for recreation and tourism.

We also need to monitor forests for the sake of fire management. For years, forest fires were suppressed, but this led to an overload of dead timber and brush, which made forests into "tinderboxes" (Ferguson 2012). When fires accidentally broke out, they were hard to control and destroyed nearby homes. Now forest scientists understand that occasional small fires are an important part of maintaining a healthy forest (Ferguson 2012). Monitoring forests helps us to predict what areas are at risk for bad fires and to try to minimize the risks through management. For more information about what the USDA's Forest Service is doing in this area, see their Wildland Fire and Fuel research website at *www.fs.fed.us/research/wildland-fire.*

Forests also provide many ecosystem services—habitats for all kinds of wildlife, insects, and migrating birds, and a diversity of plants, mushrooms, and other life. Forests are integral to the quality of our air as well. Leafy trees absorb nitrous oxides, sulfur dioxide, carbon monoxide, and ground-level ozone, which contribute to air pollution. On the other hand, air pollution damages forests, and it is important to try to understand and lessen this damage.

Finally, trees are important to the global carbon cycle and therefore affect our predictions of global climate change. Globally, forests soak up 8.8 billion tons of carbon dioxide each year (Pan et al. 2011); reducing deforestation could help as a strategy to mitigate the effects of global warming (IPCC 2007). A possible extension of this activity would be to use a carbon calculator such as the one on this USDA Forest Service website to estimate carbon from individual trees (*www.fs.fed.us/ccrc/tools/ctcc.shtml*).

A forest scientist's many names

Dendrologists study trees and woody plants.
Forest ecologists study the relationships among forest species or the effects of management or pollution on the ecosystem.
Tree physiologists study how trees work on the inside: their health or how they function as an organism.
Foresters manage forests and use science and math to do so.

grow wider crowns. The width of the crown can tell us how much competition the tree has from neighbors (packed close together, the tree cannot spread out its branches to capture more sunlight). Ask students to come up with a method for measuring the width of a tree crown. One possibility is to have students stand directly below the tips of the outermost branches and stretch a tape measure in between. Ask your students what they should do if the crown is irregular, which happens commonly. Let them wrestle with figuring out a reasonable method. Measuring two cross-sections of the crown at the largest and narrowest points and averaging them can give a reasonable overall estimation.

Another useful width measurement is the diameter of the tree trunk. Foresters prefer not to cut down or bore holes in trees if they do not have to, so they measure width by wrapping a tape around the circumference of the trunk. In the next part of the activity, explore with your students how this is possible.

Making a tape to measure trunk width

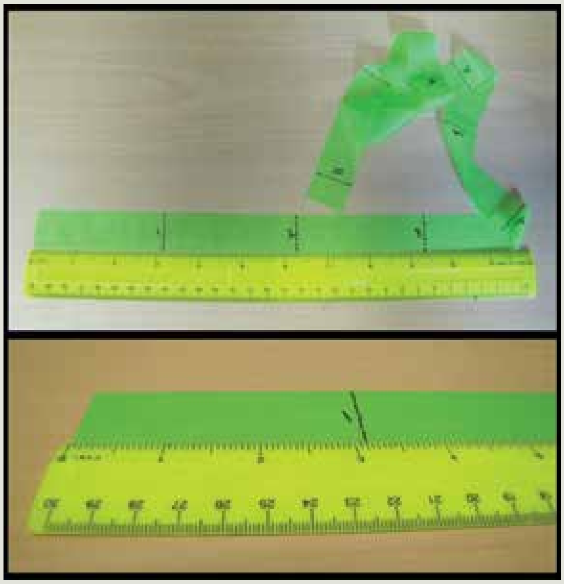
Help your students discover how to make their own tape to measure width by exploring the equation $C = \pi D$ (circumference = $\pi \times$ diameter) using Part B of the Activity Worksheet. Sometimes students are so used to the drill of substituting numbers into this equation that they have forgotten that π is a useful ratio. Students should pay special attention to this question: "For every 1 in. increase in diameter, how many inches does the circumference increase?" We suggest pointing out to students that when they think in this way, they are using mathematical modeling. You could then ask students to think about and share ideas for a way to invent something that, when wrapped around the circumference of a tree or something else round, automatically provides the diameter. At this point, distribute the flagging tape, markers, and rulers (see Figure 4 for a materials list). Have students measure and mark off increments of approximately 3.14 in. as illustrated (Figure 5). The number π is irrational, meaning that its decimal expansion never ends. It starts out 3.14159…, but here we will estimate it to be about 3 1/8 in. It helps to walk around the class with an already-made tape and compare yours to theirs to make sure students are on the right track.

DBH, or diameter at breast height, tapes, are commercially available to foresters as a tool to use

FIGURE 4 Materials for making DBH tapes

- Ruler
- Permanent marker
- 3 ft. of plastic flagging tape (sold in rolls at most hardware and home improvement stores; one roll of 200 ft. is about $2)
- Measuring tape

FIGURE 5 Making DBH tapes



in the field. DBH tapes made for forest scientists will be more precise than the ones students create, but in principle they work exactly the same way. Also, why do we suggest using inches, given the widespread use of the metric system in science? Engage students in a discussion of how the forest industry in the United States uses English units; this is commonly how you will see this measurement reported. Encourage students to convert their measurements into the metric units that are used by forest scientists around the world (using centimeters), or make a metric version of the DBH tape instead.

Another interesting point to discuss about DBH measurement is the importance of measuring DBH at the defined "breast" height of 1.37 m (4.5 ft.) above the base of the tree on the uphill side. Measuring at a significantly higher or lower level than this will give different results, because a tree trunk tends to swell at its base to support the great weight of a tree. Therefore, a defined height is necessary for consistency when recording data. Have students discover the need for a standard height by asking two students of very different heights (or yourself, if you are much taller than your students) to take the measurement of the same tree and compare results. If unable to go outside, you might have them try to take the measurement of each other—just like trees, people are also different widths at different heights, at the head, waist, and shoulders. They will realize immediately the need for a standard decision about where to measure when trying to compare student widths that have been measured at different heights. After explaining the standard height, students will notice that 4.5 ft. may be more like eye height to many of them, rather than breast height. Years ago, the definition came about because 4.5 ft. was about breast height for an average adult male. But forest scientists, like trees, come in various heights, and now there are many female forest scientists, too.

Students can use their DBH tapes to collect data on a stand of trees in their school yard or elsewhere. If this is to be assigned as homework, make sure to send a note home for parents to sign saying they will supervise for safety considerations. Data can be collected in tables in science notebooks, summarized using mean, median, and mode, and displayed in graphs or histograms. Students can use these histograms or measures of central tendency to draw informal comparisons of stands of trees in different places. It should be noted that the DBH tapes students use will not be very precise (i.e., their answers/measurements may typically be off by +/- 0.2 in.). In the Activity Worksheet, students are asked how they might improve the precision of their measurements. Precision can be increased by using real DBH tapes from forestry supply companies or measuring the circumference with cloth measuring tape and converting to tree diameter using $C = \pi D$. Students could make an investigation out of comparing the accuracy of their homemade DBH tapes to that of this other method.

Scaling up: Modeling a forest

Now that students have learned about measuring trees and hopefully taken some data, the next step is to urge them to explore how they can use these measurements, or new ones, to compare two forests that look very different. You might transition by sharing with them some pictures of forests, such as those in Figure 6, and ask students to describe the differences in words. How could they describe those same differences with numbers?

Students will observe that one forest in Figure 6 has many more trees closer together, while in the other, trees are more spread out. Ask students how they could describe these differences with numbers. One measurement foresters use to characterize this difference is stand density, or the number of trees per unit area. Students can practice calculating it from the example given in Figure 7. Then, together they could calculate the stand density of the class. Have student volunteers measure the dimensions of the classroom to find the area of their "stand," then have another volunteer count the number of people (or "people trees") in the room. Use these numbers to calculate your class's stand density.

Students may also observe from Figure 6 that some of the trees are thin and spindly, while some are much thicker. To cap off the lesson, calculating the basal area of a forest stand is a great way to tie together stand density with the DBH data taken in the first part of the lesson. Basal area takes into account the thickness of the trees—it is the portion of a plot's area that is occupied by the cross-sectional area of all tree trunks at breast height. A way to help students visualize this is to show them an image like the one in Figure 7 and explain that it is the area that is represented by brown circles divided by the total area of the plot.

Students can calculate basal area from their data in three steps: (1) converting each tree's DBH measurement to cross-sectional area by using the equation $A = \pi r^2$ (r = radius; recall that $D = 2r$); (2) summing the cross-sectional area of all of the trees in their plot; and (3) dividing their sum by the total area of the plot itself. If comparing two different stands, it is best to use plots in each stand of equal area. Students could also use graph paper to make a scale diagram (another kind of model) to show their forest stand and then calculate the basal area to further mathematically model their forest. Have students work alone or in pairs through Part C of the Activity Worksheet as a formative assessment of their understanding.

Student-proposed investigations

In designing their own investigations (Part D of the Activity Worksheet), students exercise an authentic scientific practice skill by creating their own testable questions; such questions can come from discussing why forest scientists are already measuring and monitoring forests. This portion of the worksheet could serve as a hypothetical exercise or be carried out in a forest or stand of trees to which you have access. This section is designed to organize students' thoughts and may be used as a formative assessment tool. A first step might be to complete the activity once as a class discussion, then have students complete it on their own for a different forest. The activity may be completed by groups or by individual students who then share and revise with a partner. The first couple of questions are to help students think of a concrete example of a forest that is meaningful to them and that they may know some information about, or that they can research online. It may be one that the class has already visited or discussed. The objective is to help students make the connection between what they have learned about modeling and how it is useful to answer scientific questions. Encourage students to refine their questions into testable ones by asking them, "How would you test that? What tools would you use? What data would you collect?"

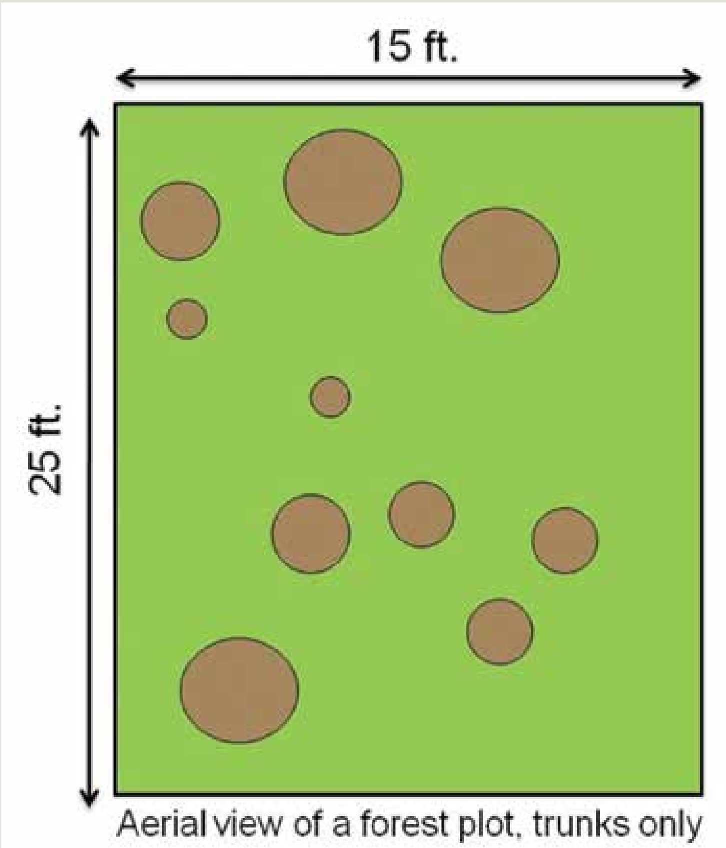
FIGURE 6 Different forest types: (A) Chequamegon-Nicolet National Forest, Wisconsin. (B) Mendocino Pygmy Forest, California.

PHOTO BY JASON STURNER

Ideas for extensions

Ideas for extending this activity are many and can extend beyond the science classroom. In addition, Project Learning Tree (see Resources) has

FIGURE 7 A drawing model of a stand of trees from above, with branches missing



an entire K–12 curriculum designed around what can be learned from trees and forests. Below are some of our ideas:

- If learning more about the tree as an organism is a part of your curriculum goals, tree cookies are excellent for further examinations into the structure of tree trunks, as well as the stories they tell. Students can count rings to determine tree age as well as identify the structure and learn the functions of bark, phloem, xylem (heartwood and sapwood), cambium, and pith.
- Rather than show a video, invite a forest scientist to come speak to your class.
- Deforestation is a problem affecting many countries around the world. In addition, lumber is an economically valuable resource. Consider a social-studies tie-in by involving a colleague on your teaching team.
- The study of trees has inspired many interesting books. Consider making a language-arts connection by having students read *Reading the Forested Landscape: A Natural History of New England* by Tom Wessels or *The Wild Trees: A Story of Passion and Daring* by Richard Preston.

Summing it up

Throughout this activity and at the end, it may be useful to have students reflect on how what they are doing is mathematical modeling, an important scientific practice skill and mathematical tool. This could be addressed through classroom discussion, or students might think-pair-share to describe the patterns they find in their data. In this way, the lesson can emphasize the modeling and mathematical thinking skills that are important in both the *Framework* and the *Common Core State Standards, Mathematics.*

At the end of this activity, students should be able to describe reasons why it is useful to measure and moni-

tor forests; understand that measuring and monitoring forests involves modeling; and apply taking data such as DBH, or calculating stand density or stand basal area from data, to design investigations to answer basic scientific questions of their own creation.

All of these activities will be twice as fun if students have the opportunity to go outside on school grounds to test out what they have learned on real trees, but we understand that this is often not possible. When we are limited to doing this activity indoors, we use tree cookies, which are round, cross-sectional slices of tree trunks (anything circular will work as a model tree cookie). To create your own classroom set, these can be purchased reasonably, or instructions are available online to make your own (see NIMBioS in Resources).

Forest scientists use both science and mathematics in trying to understand our world. Modeling a forest using mathematics is an engaging way to get students thinking about how data and math can be used to represent and understand phenomena. It is also an authentic way to integrate standards across disciplines to promote deeper understanding. When students wrap measuring tape around their trees, you might smile when you see how it looks like they are giving those trees a hug. ■

Acknowledgment

The tree clip art used to create Figures 2 and 3 is from Karen Hatzigeorgiou's online clip-art collection. She curates these public-domain images and makes them available for noncommercial use (see *http://karenswhimsy.com/tree-clipart.shtm*).

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Resources

How to use a clinometer—*http://elms.smcps.org/student-tutorials/great-trees/using-a-clinometer-to-measure-height*

NIMBioS education module: Measuring a forest—*www.nimbios.org/education/measure_a_forest*

Project Learning Tree—*www.plt.org*

Schoolyard clinometer—*www.state.nj.us/dep/seeds/syhart/clinom.htm*

Forest and tree images

Encyclopedia of life—*www.eol.org*

Forestry images—*www.forestryimages.org*

Videos of ecologists

Changing forests (a forest ecologist talking about her work studying forests and climate change)—*www.nimbios.org/videos/vid_moran*

Wild-plant harvesting (an ecologist talking about his work studying sustainable harvesting from plants such as trees)—*https://www.nimbios.org/videos/vid_gaoue*

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ACTIVITY WORKSHEET: MODELING A FOREST

Part A: Modeling a single tree

1. Trees come in all shapes and sizes. Look at trees outside the window or images of trees provided by the instructor. How does the structure of different trees vary? List three different ways. (Hint: Don't forget that trees are three dimensional; what does that mean?)

   (a)

   (b)

   (c)

2. Can trees that are the same species look different? What are three possible reasons why?

   (a)

   (b)

   (c)

3. For question 1 above, how could you measure and describe with numbers the ways that you listed that trees may vary? Describe what tools you would need and how you would use the tools.

   (a)

   (b)

   (c)

Part B: Making a DBH tape

Foresters use a special tool called DBH tape to measure the diameter of tree trunks. DBH stands for diameter at breast height. DBH tape is wrapped around the outside of a tree trunk to learn the tree's diameter.

1. If you were to use a normal measuring tape and wrap it around the trunk of a tree, how might you figure out the tree's diameter? Explain.
2. Imagine a cross-section of a perfect tree trunk, if the trunk was cut horizontally. Draw what it would look like. Draw and label the diameter. Next, draw and label the circumference.
3. Practice using $C = \pi D$ (where C is circumference, D is diameter, and π is ~ 3.14) with a calculator.

   (a) If D = 2, C = ___

   (b) If D = 6, C = ___

   (c) If D = 1, C = ___

   (d) If C = 3.14, D = ___

   (e) If radius (R) = 3, D = ___

   (f) For every 1 in. increase in diameter, the circumference increases ___ inches.

4. If possible, find the diameter of your tree, tree cookie, or another round object using a regular ruler and record your measurement. If not possible, put "N/A" here and explain why.
5. Find the diameter of your tree, tree cookie, or other round object using a DBH tape and record your measurement.
6. Are the answers to questions 4 and 5 the same? Why or why not? How could you improve your precision?
7. Find the diameter of your head using a DBH tape.
8. Is measuring DBH at a standard height important? Why or why not?
9. What is mathematical modeling? Explain how the DBH-tape tool uses mathematical modeling to find tree diameter.

Part C: Scaling up: Modeling a forest

1. Below is an equation for modeling forest stand density. Explain in your own words, without using numbers or equations, what stand density is.

   $$\text{Stand density} = \frac{\text{number of trees}}{\text{area of stand}}$$

2. What might affect a forest's stand density?
3. Calculate stand density for the example plot provided by the teacher.
4. Imagine that all of the students in your class are trees, and the classroom is your forest plot. What is the stand density of your student-forest?
5. Below is an equation for modeling a single tree's basal area. Explain in your own words, without using numbers or equations, what tree basal area is.

   $$\text{Basal area of tree} = \pi \left(\frac{\text{DBH}}{2}\right)^2$$

6. Below is an equation for measuring the basal area of a stand of trees. Explain in your own words, without using numbers or equations, what stand basal area is. Note: N is the total number of trees in a stand.

   Basal area of stand with N trees = Basal area of tree 1 + . . . + Basal area of tree n

7. Practice mathematical modeling of an example stand. Below are tree DBH data collected from a sample plot of 100 m^2. Use a calculator to fill in the tree basal areas and the stand basal area. Also, calculate the stand density.

DBH (cm)	Tree basal area (cm^2)
5	
11	
4	
8	
7	
9	
6	
8	
6	
2	
4	
Stand basal area	

Stand density:

Part D: Measuring forests: Propose your study

Describe a forest or group of trees that you have heard of, have visited, or that is near your home or school.

List three reasons why one might want to measure or monitor this forest or group of trees.

(1)

(2)

(3)

Ask a question you have about the forest you have described, one that can be tested by measuring the trees using the methods discussed in class. (Hint: You might use the answers to the previous question to guide you!)