Solving Discrete AMs with MDPSolve

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Example – Pests revisited (see Simple_pestsAM1.m)

Recall that:

Pests are classified into 3 levels: low, medium and high: $S \in \{1,2,3\}$ (so $\#S = 3$)

The site can be left alone or treated: $A \in \{0,1\}$ (so $\#A = 2$)

Damage costs are $D = [0;\ 5;\ 20]$ for the 3 states

Treatment cost is $C = 10$

The discount factor is $\delta = 0.95$

There are 6 possible state/action combinations $(\#X = 6)$

With action (A) first (very important to keep ordering consistent) this is

$$X = \begin{bmatrix} 0 & 1 \\ 0 & 2 \\ 0 & 3 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$$

The reward function can be written as $R = -[D; D + C]$ (this is a $\#S \times \#A$ matrix)

Written as the negative of cost (recall that we are maximizing)

Transition Probabilities

Probabilities with no action remains the same

$$P_0 = \begin{bmatrix} 0.65 & 0.15 & 0.05 \\ 0.25 & 0.40 & 0.20 \\ 0.10 & 0.45 & 0.75 \end{bmatrix}$$

Probabilities with treatment are not well known

Suppose we have a pessimistic and an optimistic assessment:

$$P_{1p} = \begin{bmatrix} 0.7 & 0.25 & 0.15 \\ 0.3 & 0.45 & 0.30 \\ 0 & 0.30 & 0.50 \end{bmatrix}$$

$$P_{1o} = \begin{bmatrix} 1 & 0.75 & 0.65 \\ 0 & 0.25 & 0.30 \\ 0 & 0 & 0.05 \end{bmatrix}$$

Discrete Belief Grids

A belief state with m alternative models is an m vector of non-negative numbers that sum to 1

For example: [0.25 0.5 0.25]

A discretization of the belief state is a grid of values with each row satisfying the summing up condition

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

MDPSolve has a function **`simplexgrid`** that creates such a grid.

The syntax **`b=simplexgrid(cat,inc,sum)`** produces

an evenly spaced grid of values on a simplex for **`cat`** variables

each variable has **`inc+1`** values and

the values on each row sum to **`sum`**.

The above example is obtained using **`b=simplexgrid(3,2,1)`**

Note that the grid values are arranged in lexicographic order (this is important for correct interpretation).

Using amdp

The MDPSolve function facilitates the specification of discrete AM models

The basic syntax is

```
Ix = getI(X,svars);
[b,Pb,Rb,Sb,Xb,Ixb ]=amdp(inc,P,R,S,X,Ix);
model = struct('P',Pb,'R',Rb,'Ix',Ixb,'d',delta);
results = mdpsolve(model);
```

In contrast to an ordinary MDP model here we define **`P`** as a cell array with m transition matrices
In our example we have **`P = {[P0 P1p],[P0 P1o]};`**
`amdp` also uses the index vector **`Ix = getI(X,svars);`**

It can also be useful to solve each of the alternative models separately as ordinary MDPs

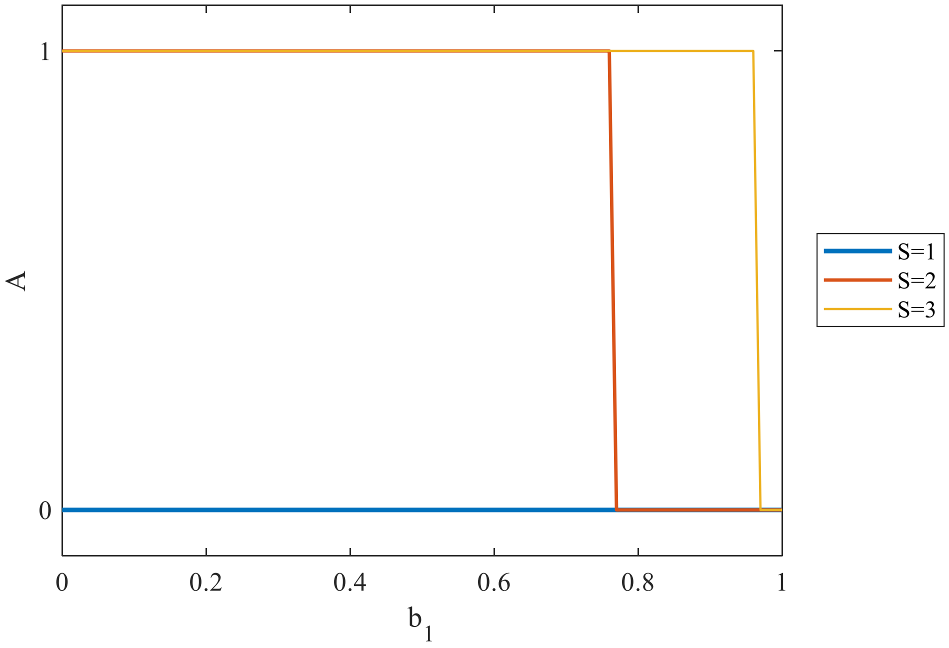
	Optimal action		Value	
S	pessimistic	optimistic	pessimistic	optimistic
low	0	0	-221.8	-113.5
medium	0	1	-236.7	-125.8
high	0	1	-261.0	-142.7

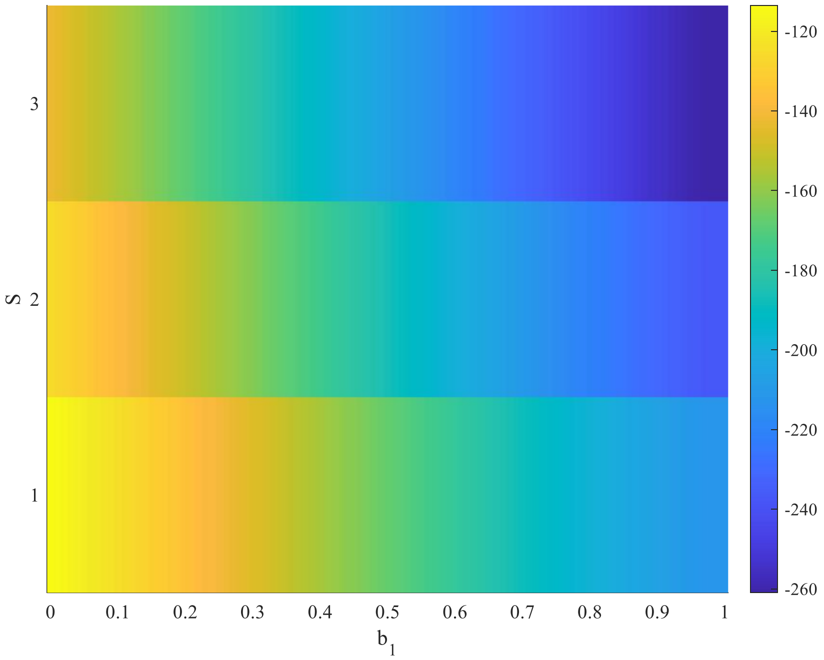
With the pessimistic assessment we give up – it is not worth treating
With the pessimistic assessment we treat in both the medium and high population levels

Solution to the pest problem

The optimal action is to treat on both the medium and high population states unless we have strong belief in the pessimistic model ($b_1 \geq 0.77$ for $S = 2$ and $b_1 \geq 0.97$ for $S = 3$)

The value function is plotted on the right: value declines as the state increase and as b_1 increase (hopefully this in intuitive)





Caveats

The discrete AM framework here defines alternative models using alternative transition matrices
and updates beliefs using only the new state values as information

Another possibility is that the response of performance variables are uncertain

Martin, et al. considered a case in which
the fledging success of oystercatchers was a performance variable
the state was the size of the predator population and
the uncertainty was the response of fledging success to the predator population

Other information might also be useful

Suppose that we are uncertain about fecundity; direct observations on young per parent is more informative than overall
future population which might involve uncertainty about survival and harvest