

# **Single-image molecular analysis for accelerated fluorescence imaging**

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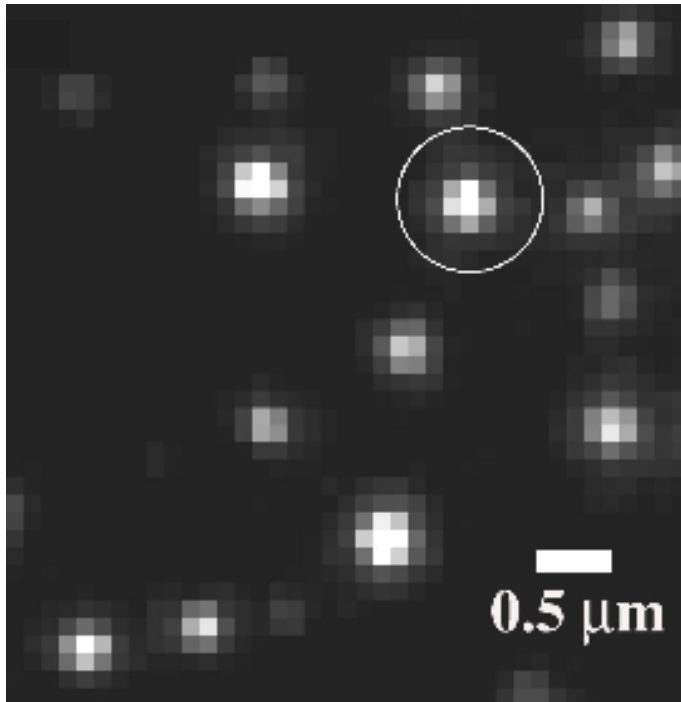
# Outline

- 1. Current single-molecule localization, separation, and dynamic measurement methods and challenges**
- 2. Use single-image molecular analysis (SIMA) to determine**
  - a. Axial-localization precision**
  - b. Separations of unresolved molecules**
  - c. Diffusion coefficients of proteins in free solution**
- 3. Applications**
  - a. Intraflagellar transport particle dynamics – BBSome and IFT**
  - b. Membrane-binding proteins –UgtP**
  - c. Photosynthesis - Phycobilisome**

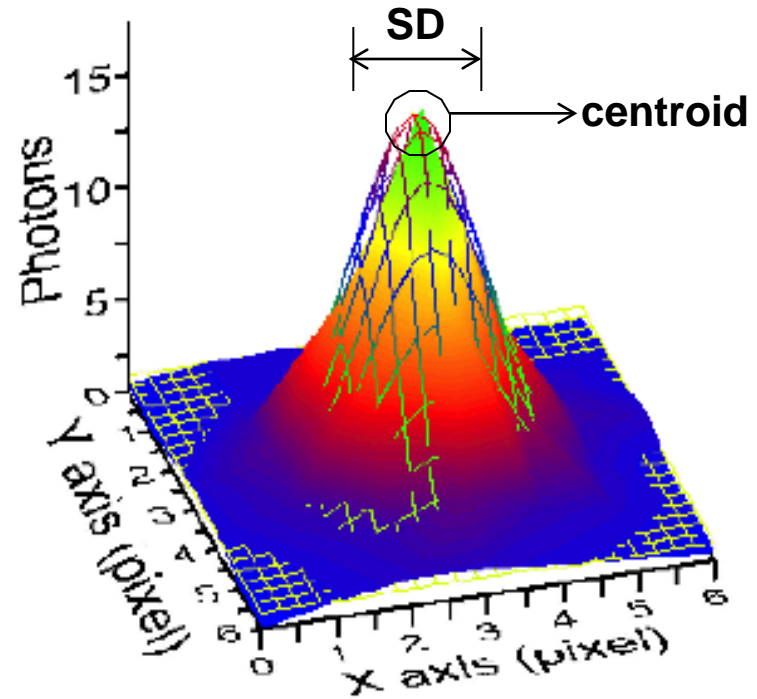
# I. 3D Localization

Gaussian-approximated point spread function (PSF):

- Centroid → Lateral location,  $x$  and  $y$
- Standard deviation (SD) → Axial location,  $z$

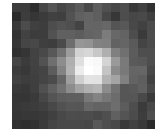


GFP, TIRF image

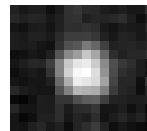


# Axial location measurement, $z$

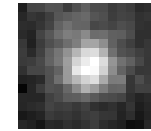
PSF Standard deviation,  $s_{x,y}$ , determines the axial location,  $z$



- 400 nm

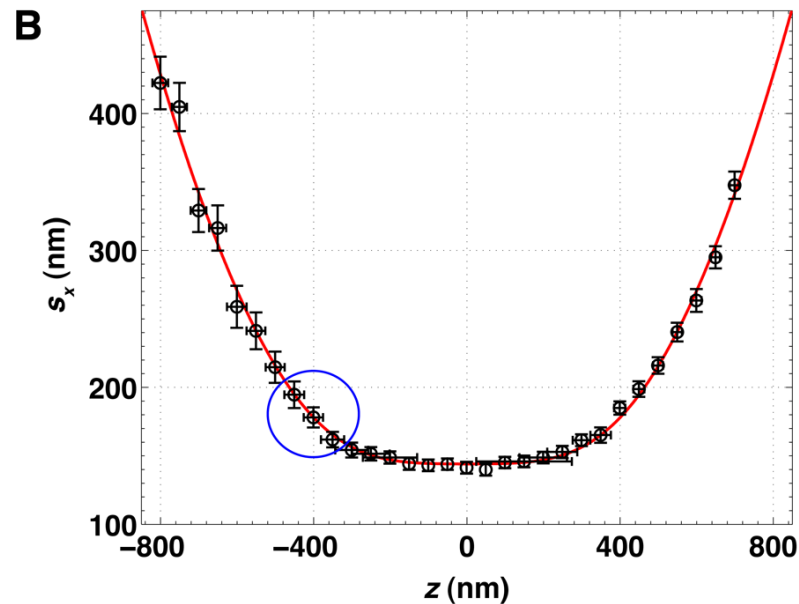


$z = 0$



400 nm

Phycobilisome protein axial location (relative to the focal plane)



$$s(z) = s_0 \sqrt{1 + \left(\frac{z}{d}\right)^2 + b \left(\frac{z}{d}\right)^4}$$

$s_0$  = SD at focus,  
 $d$  = "imaging depth"  
 $b$  = fitting parameter

# Localization precision

Precision in x-direction:

$$\sqrt{\langle(\Delta x)^2\rangle} = \sqrt{\frac{2(s_{0x}^2 + \frac{a^2}{12})}{N} + \frac{8\pi(s_{0x}^2 + \frac{a^2}{12})^{3/2}(s_{0y}^2 + \frac{a^2}{12})^{1/2}(\sigma_b^2 + \langle b \rangle)}{a^2 N^2}}$$

**a** - pixel size

**S<sub>0x/y</sub>** - standard deviation (SD) in x/y direction

**N** - number of photons

**σ<sub>b</sub>** - background noise standard deviation

**⟨b⟩** - background noise mean

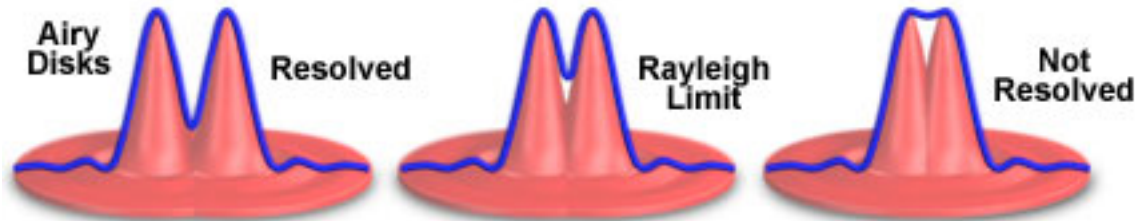
$$\sqrt{\langle(\Delta x)^2\rangle} \approx 7 \text{ nm for 1000 photons}$$

- **Problem, there is no axial precision expression, Δz**

**Repeated measurements are used to obtain Δz (seconds of imaging)**

## II: Separation measurements

### Airy Disk Separation and the Rayleigh Criterion

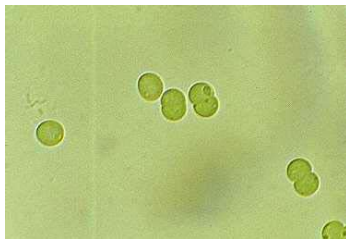


Rayleigh criterion separation =  $0.61\lambda/NA \approx 240 \text{ nm}$

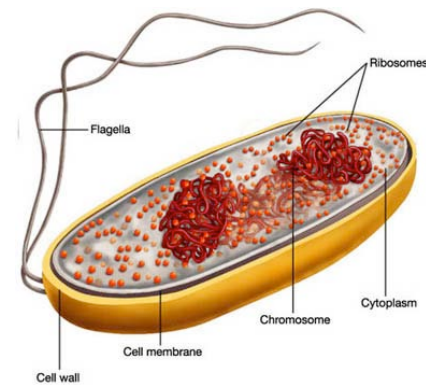
$\lambda$  = wavelength = 550 nm

$NA$  = numerical aperture = 1.49

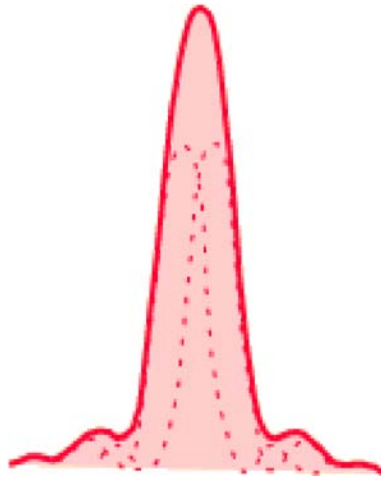
Synechocystis (3  $\mu\text{m}$  cyanobacteria)



**E. coli**



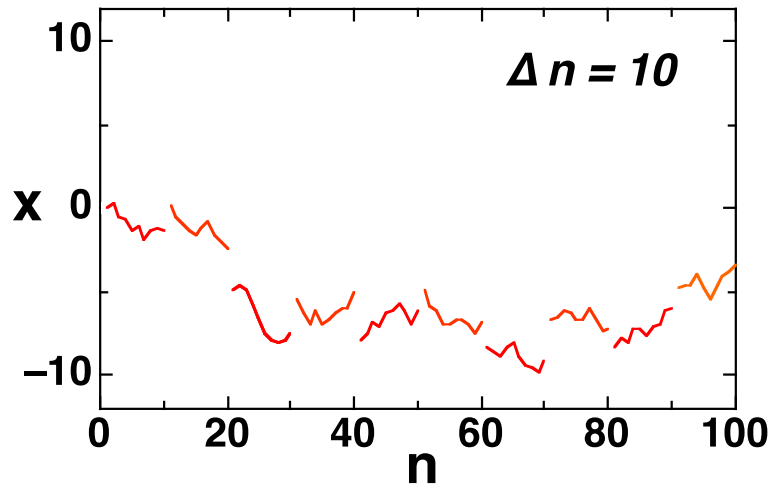
# Disadvantages of current methods



1. Photobleaching of the molecule, SHRImP
2. Multiple color, SHREC
3. Photoswitchable fluorophores
4. Centroid measurements, long measurement time:  
> seconds

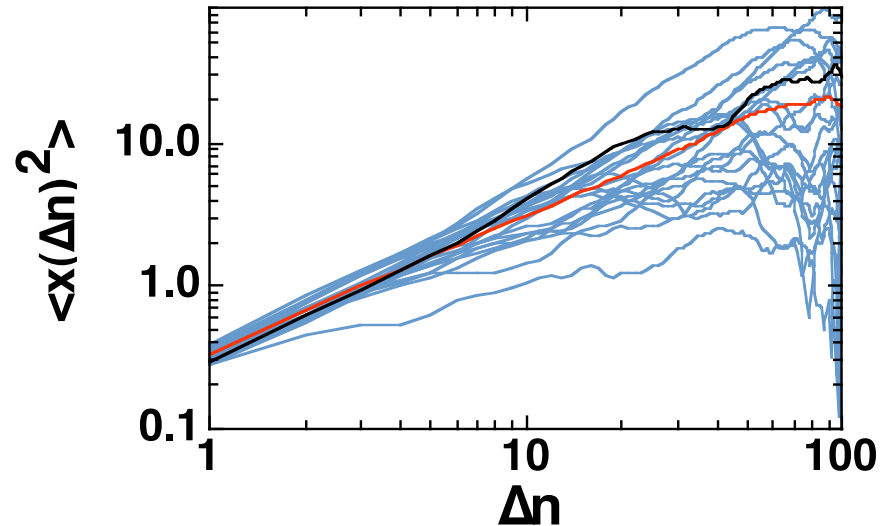
### III. Dynamics studies, single-molecule tracking

#### Simulation of individual Brownian trajectories



$$\langle x(\Delta n)^2 \rangle = 2D_1 \Delta n$$

$$x(10) = (x_{11} - x_1), (x_{12} - x_2), \dots (x_N - x_{N-10})$$



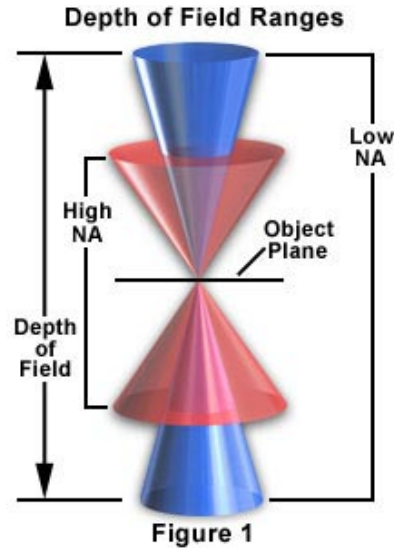
$$\langle x(\Delta n)^2 \rangle = 2D_1 \Delta n \pm \sigma(\Delta n, N)$$

$$\sigma(\Delta n, N) = 2D_1 \Delta n [(2\Delta n^2 + 1)/(3\Delta n(N - \Delta n + 1))]^{1/2}$$

$$\Delta n = 1, 2, 3, \dots, N$$



# Limitation of current single-molecule $D_3$ measurements

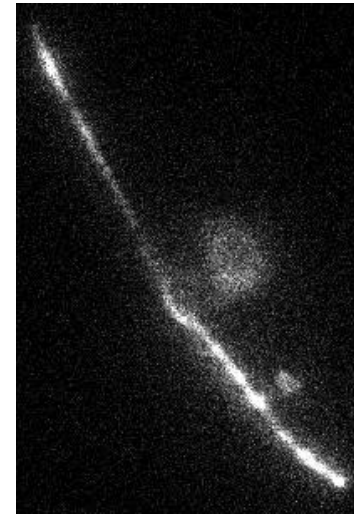


- Depth of imaging  $< 300$  nm
- A 5 nm wide GFP,  $D_3 \approx 10^8$  nm<sup>2</sup>/s, moves out of the imaging depth in 1 ms
  - $\langle x^2 \rangle^{1/2} = 300$  nm =  $(2D_3t)^{1/2} \Rightarrow t = 1$  ms
  - With 100 Hz camera imaging rate,  $D_{3,max} \approx 10^5$  nm<sup>2</sup>/s

Recent  $D_{3,max} \approx 2 \times 10^7$  nm<sup>2</sup>/s measurements require two-color labeling  
(Stefan Semrau et al., *BPJ*, 2011)

# Biological systems need fast single-molecule investigations

## Example: Intraflagellar transport - IFT



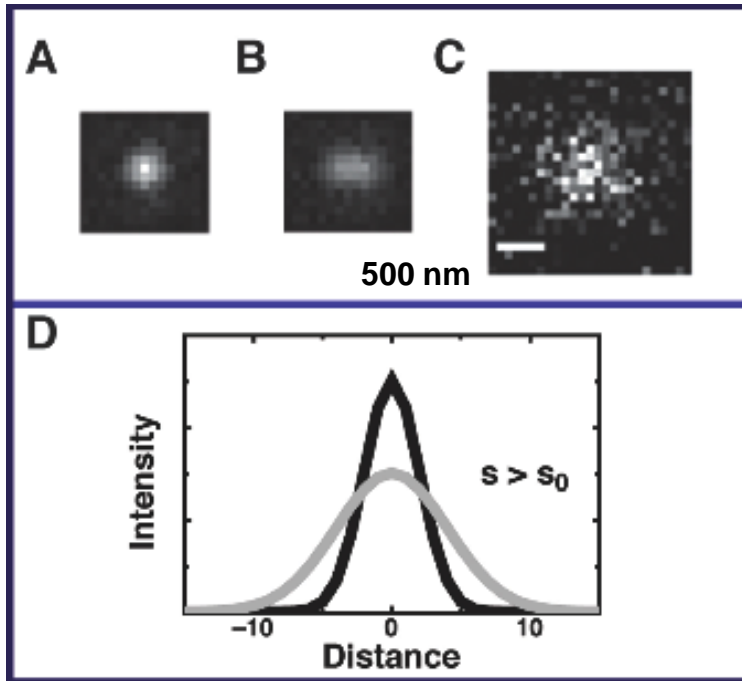
Kinesin-GFP, 2  $\mu\text{m/s}$

- IFT particles travel to the flagellar tip and back
- Carry flagellar materials
- Carry signaling proteins

Direct relevance to human disease:

such as, Bardet-Biedl syndrome & Polycystic Kidney Disease

# Solution: analyze the spatial distribution of photons



Blurred moving car



- (A) A stationary molecule
- (B) Two fluorophores separated by 237 nm
- (C) A diffusing GFP in solution (1 ms exposure time)
- (D) The SD difference.

# Compromising the spatial resolution?

**Temporal resolution = submillisecond-milliseconds = single-image exposure time**

**Spatial resolution = nanometers**

**Theoretical SD measurement error**

$$\langle \Delta s_x \rangle = \sqrt{\frac{s_{0x}^2 + \frac{a^2}{12}}{2N} + \frac{16\pi(s_{0x}^2 + \frac{a^2}{12})^{3/2}(s_{0y}^2 + \frac{a^2}{12})^{1/2}(\sigma_b^2 + \langle b \rangle)^2}{3N^2 a^2}}$$

$\Delta s_x =$  SD measurement error

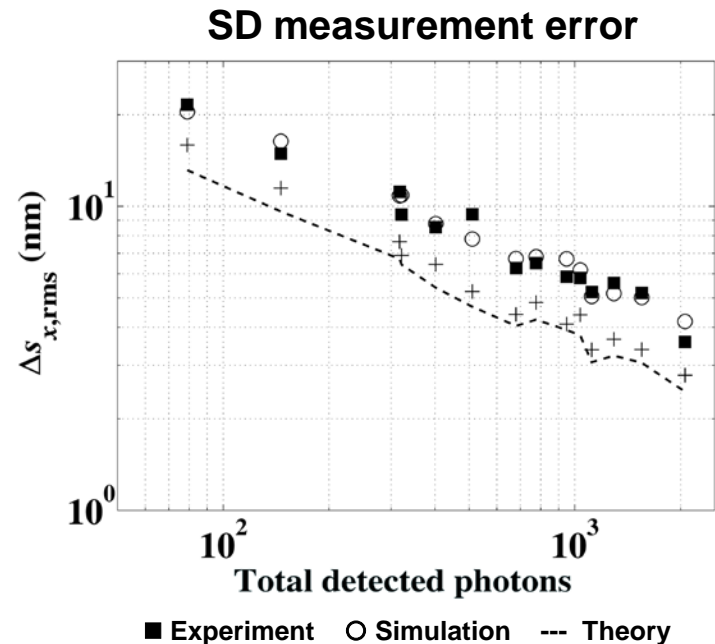
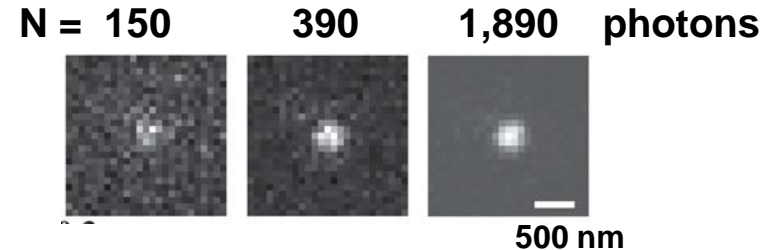
$s_0 =$  theoretical PSF SD  $\approx 120$  nm for Cy3

$a =$  pixel size  $\approx 79$  nm

$N =$  number of photons in the PSF

$\sigma_b =$  SD of background photon count

$\langle b \rangle =$  mean background photon count



# I. $\Delta z$ , axial localization precision

**Error propagation:**

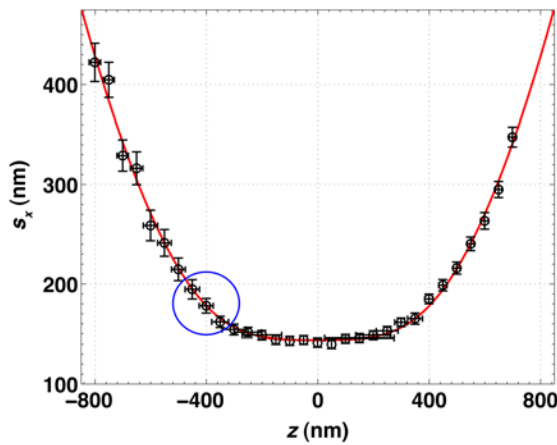
$$s(z) = s_0 \sqrt{1 + \left(\frac{z}{d}\right)^2 + b \left(\frac{z}{d}\right)^4} \Rightarrow \Delta z(s) \propto \Delta s$$

**$\Delta z$  is a function of SD,  $s$**

$$\begin{aligned} \Delta z &= \frac{sd\Delta s}{\sqrt{2s_0^2 \sqrt{\left(\frac{s}{s_0}\right)^2 - 1 + \frac{1}{4b}} \left(2\sqrt{b} \sqrt{\left(\frac{s}{s_0}\right)^2 - 1 + \frac{1}{4b}} - 1\right)^{1/2}}} \\ &= \frac{sd \sqrt{\frac{s_0^2 + \frac{a^2}{12}}{N} + \frac{16\pi(s_{0x}^2 + \frac{a^2}{12})^{3/2} (s_{0y}^2 + \frac{a^2}{12})^{1/2} (b^2 + \langle bg \rangle)}{3a^2 N^2}}}{\sqrt{2s_0^2 \sqrt{\left(\frac{s}{s_0}\right)^2 - 1 + \frac{1}{4b}} \left(2\sqrt{b} \sqrt{\left(\frac{s}{s_0}\right)^2 - 1 + \frac{1}{4b}} - 1\right)^{1/2}}} \end{aligned}$$

# Experiments agree with the $\Delta z$ expression

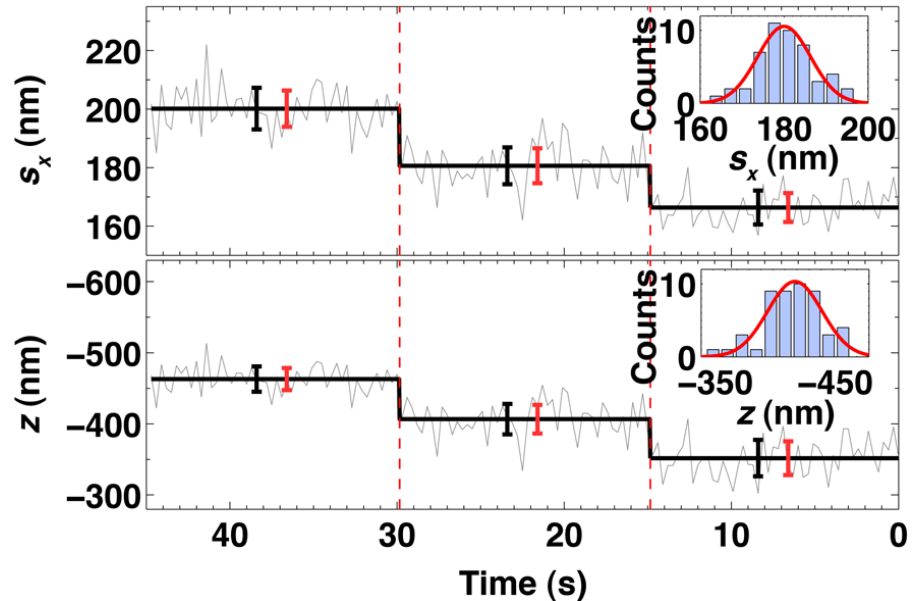
Phycobilisomes  $s$  vs.  $z$  plot  
(660 nm emission)



Error bars:

$\Delta s$  (vertical) and  $\Delta z$  (horizontal)  
at 1000 photons/image;  
 $\Delta z = 34 \text{ nm}$  at  $z = 400 \text{ nm}$

$s$ ,  $z$ ,  $\Delta s$ , and  $\Delta z$  for 20 nm  $\Delta z$  steps



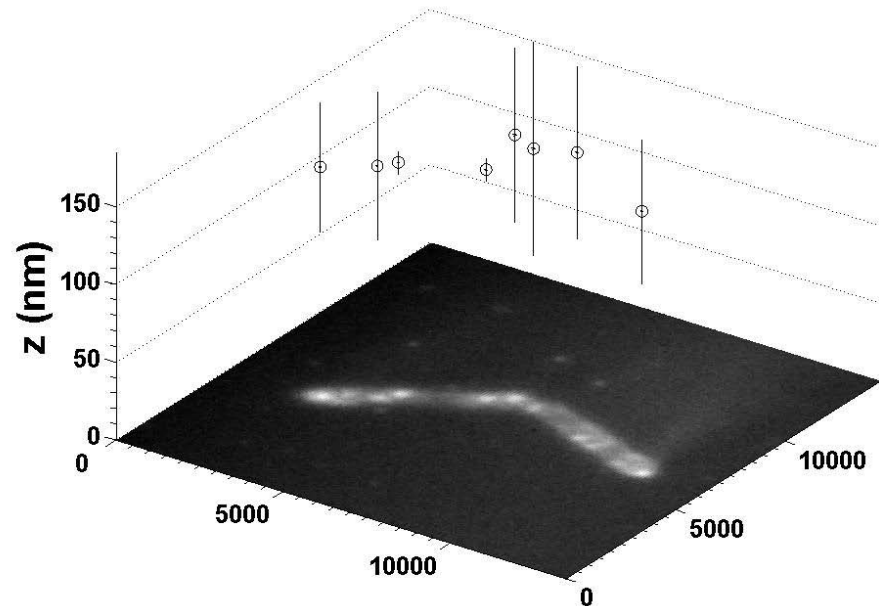
**This study allows single PSF 3D-localization measurements with precision**

# Application I: membrane glycolipid synthesis enzyme, UgtP-YFP, membrane interaction statics and dynamics

UgtP-YFP puncta in *Bacilica subtilis*

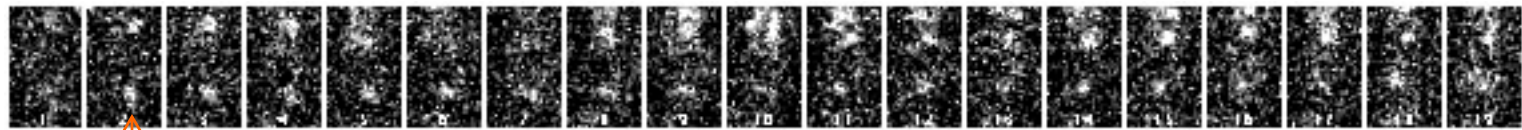


3D location and precision



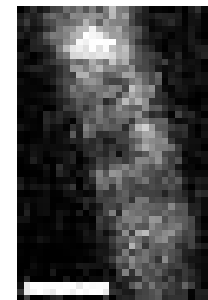
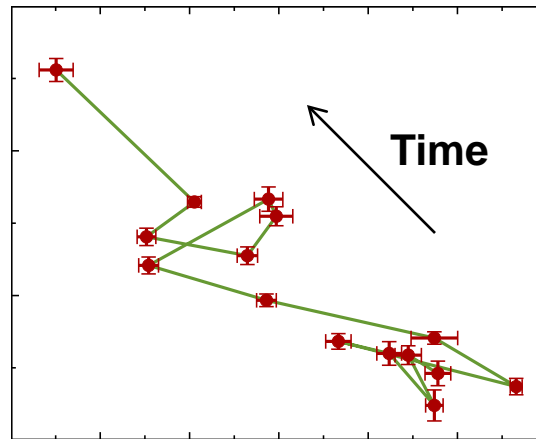
# UgtP-YFP moves along a helical path

UgtP-YFP diffusing along the membrane?



0.33 s/image

Helical path

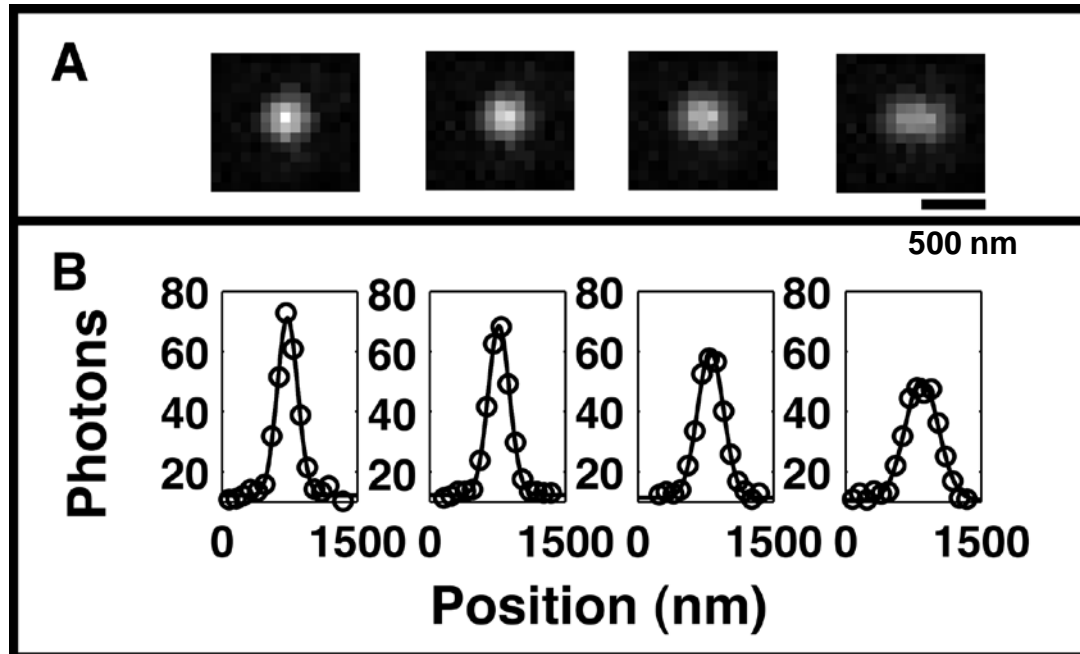


1  $\mu\text{m}$



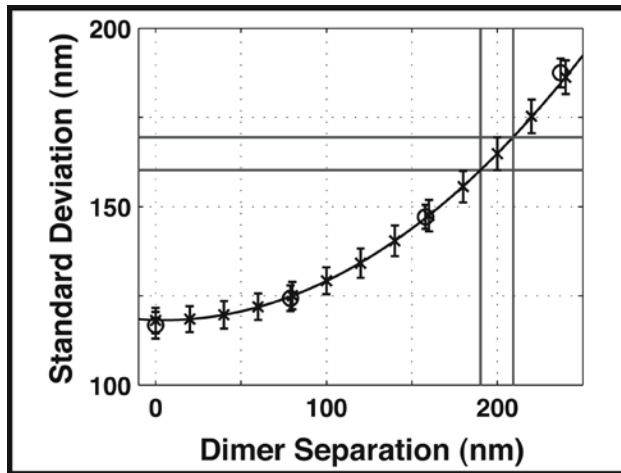
## II. Unresolved identical fluorophores; dimer separation measurements

Separation = 0 nm      79 nm      158 nm      237 nm

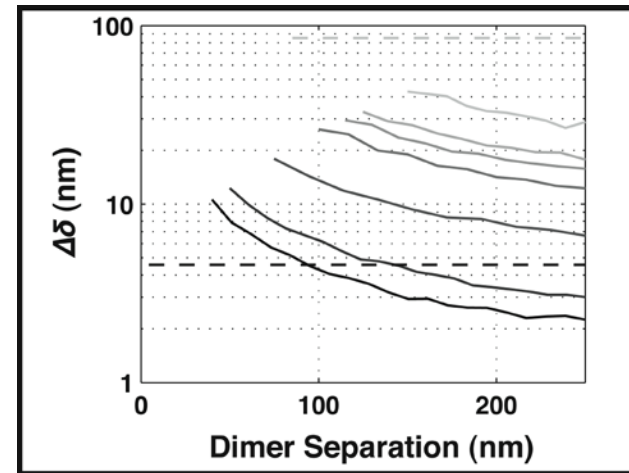


# SD measurements of dimer separations

## SD vs. separation

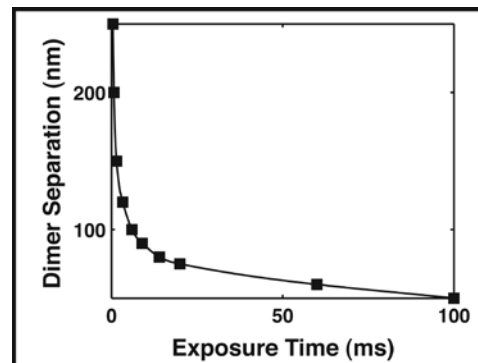


## Error to separation measurements



Top down, 150 to 20,000 photons

## Milliseconds temporal resolution



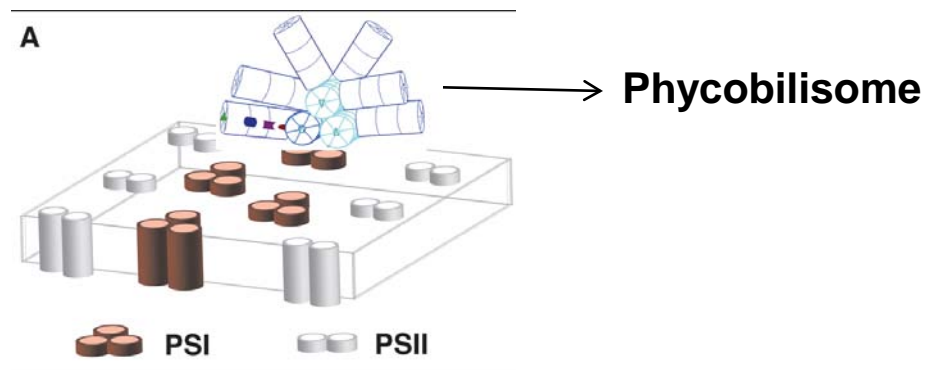
# Application II: Photosynthesis

**Cyanobacteria collect light for energy by photosynthesis;  
future substance for clean energy**

**Synechocystis (3  $\mu\text{m}$  cells)**

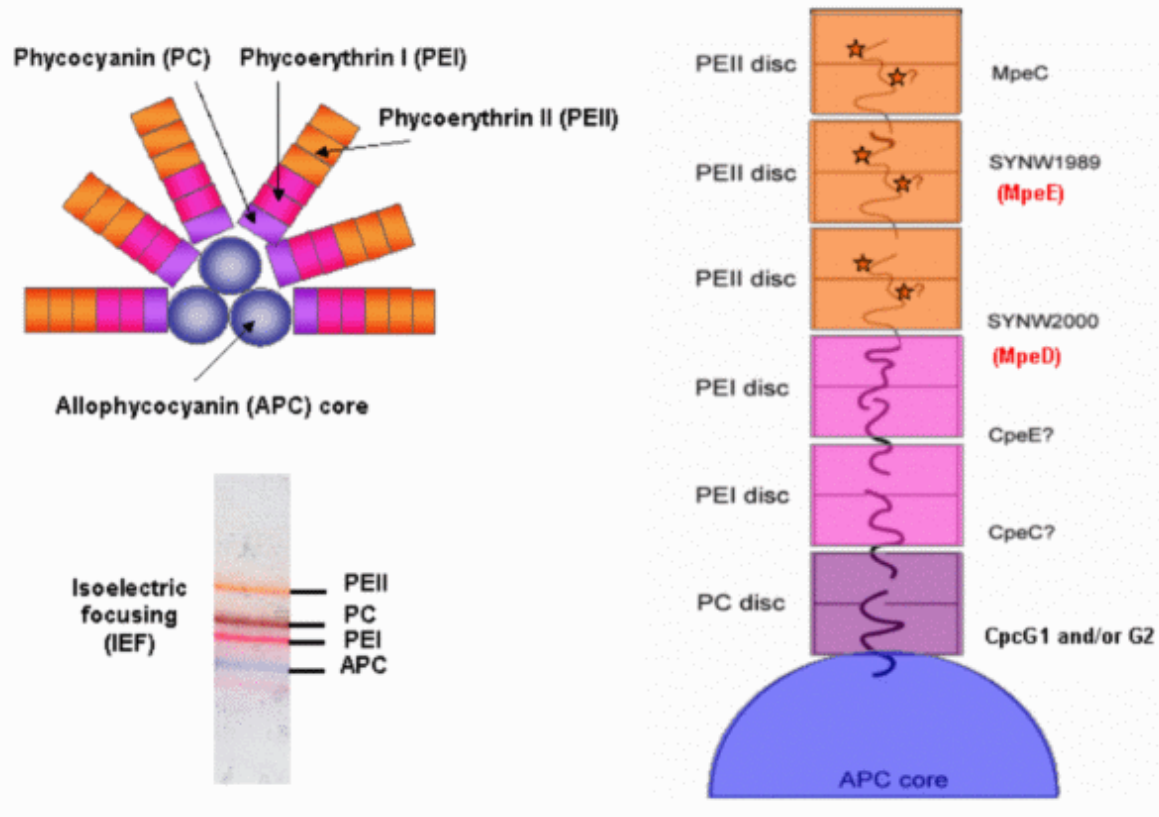


**Phycobilisomes (60 nm across),  
protein on the cell thylakoid membrane that collects light energy**



# Phycobilisome light energy transfer mechanism

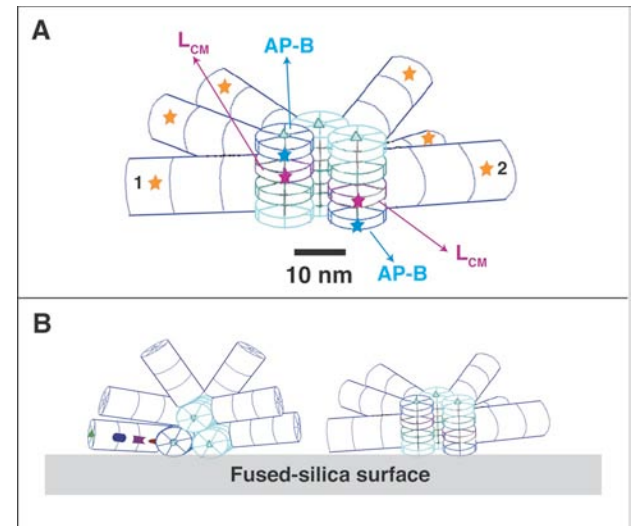
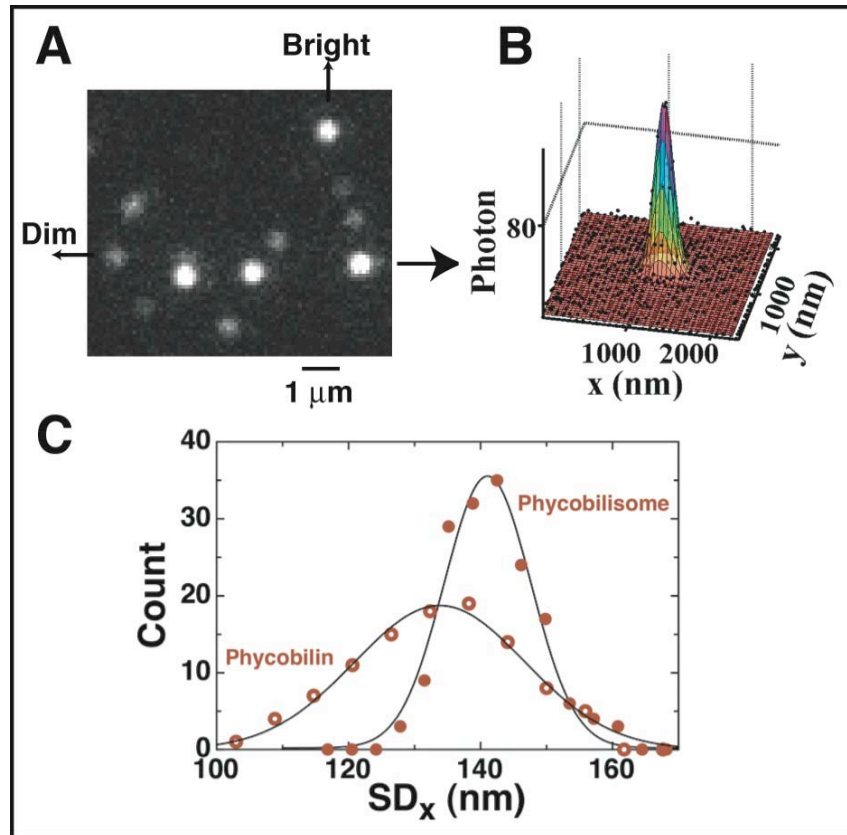
FRET through components to the terminal emitters in the core



PC and APC emission peak at 650 nm and 660 nm, cannot differentiate using conventional spectroscopy

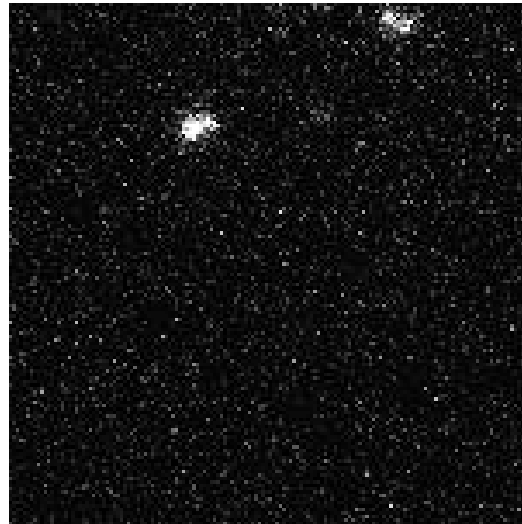
# Energy transfer efficiency < 95%

The SD of phycobilisomes is 6 nm larger than the expected value for 95% energy transfer efficiency



### III. 3D diffusing GFP in free solution

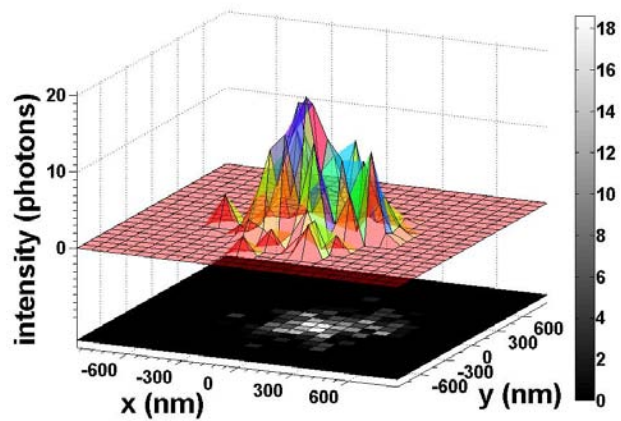
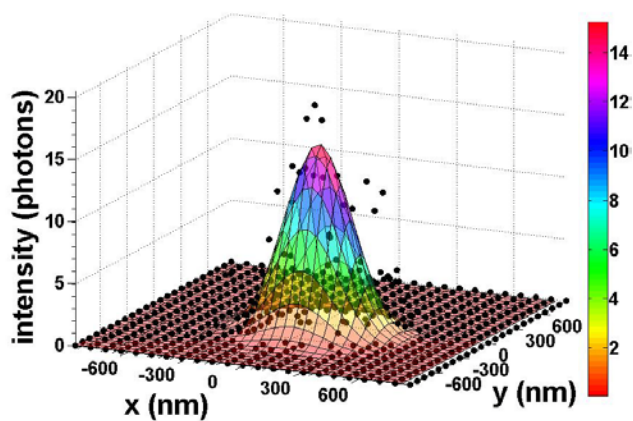
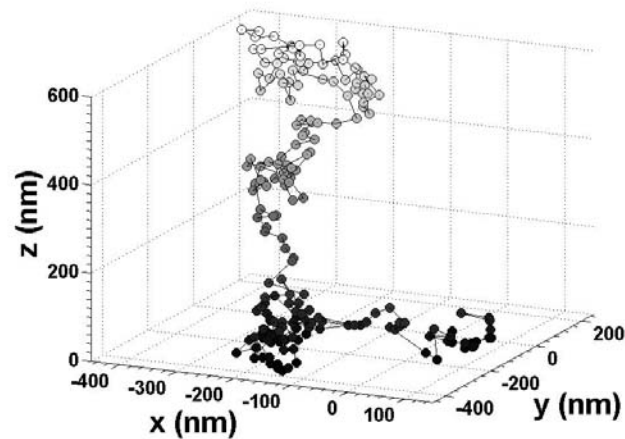
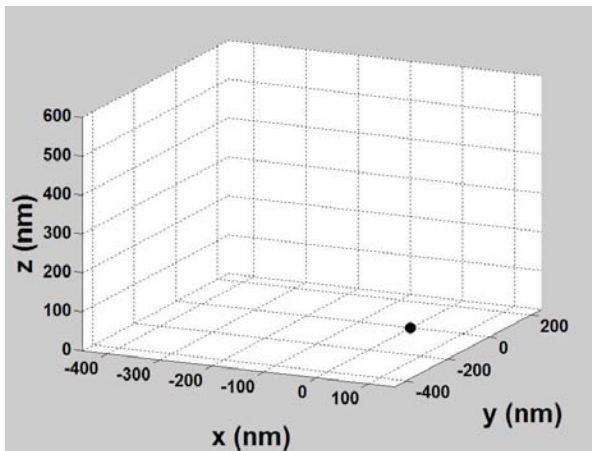
*1 ms exposure time*



12  $\mu\text{m}$

**Only one image of a diffusing molecule can be obtained**

# Simulation



## Theory: SD vs. exposure time $t$ expression

$$I(x,y) = \text{PSF}(z\text{-weighted}) \otimes \text{PWDF}_{x,y}$$

**PWDF = pathway distribution function**

$$\int_0^{\infty} \frac{1}{s_{x,z}s_{y,z}} \exp\left(-\left[\frac{x^2}{2s_{x,z}^2} + \frac{y^2}{2s_{y,z}^2}\right]\right) \exp\left(-\left[\frac{(z-\bar{z})^2}{2A_z D_3 t} + \frac{z}{z_d}\right]\right) dz \otimes \exp\left(-\left[\frac{x^2 + y^2}{2A_{x,y} D_3 t}\right]\right)$$

**Both PSF(z-weighted) and  $\text{PWDF}_{x,y}$  are Gaussian functions**



# SD vs. exposure time expression

$$I(x,y) = \text{PSF}(z\text{-weighted}) \otimes \text{PWDF}_{x,y}$$

Since both  $\text{PSF}(z\text{-weighted})$  and  $\text{PWDF}_{x,y}$  are Gaussian functions,

SD of the diffusing GFP image =  $(\text{PSF variance} + \text{PWDF}_{xy} \text{ variance})^{1/2}$

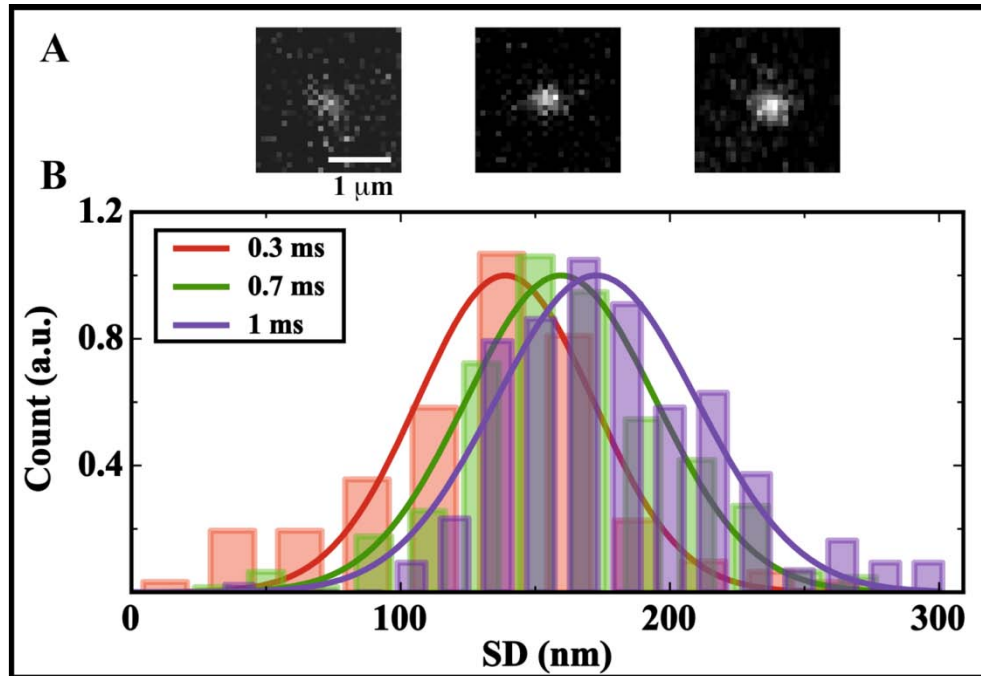
$$s_{x,y}(t) = \sqrt{s_{x_0,y_0}^2(t) + 2A_{x,y}D_3t}$$

$$\Rightarrow D_3 = \frac{s_{x,y}^2(t) - s_{x_0,y_0}^2(t)}{2A_{x,y}t}$$

Single image  $D_3$  error

$$\Delta D_3 = \frac{\delta s_{x,y}(t)}{t} \Delta s_{x,y}$$

## Experimental: SD measurements

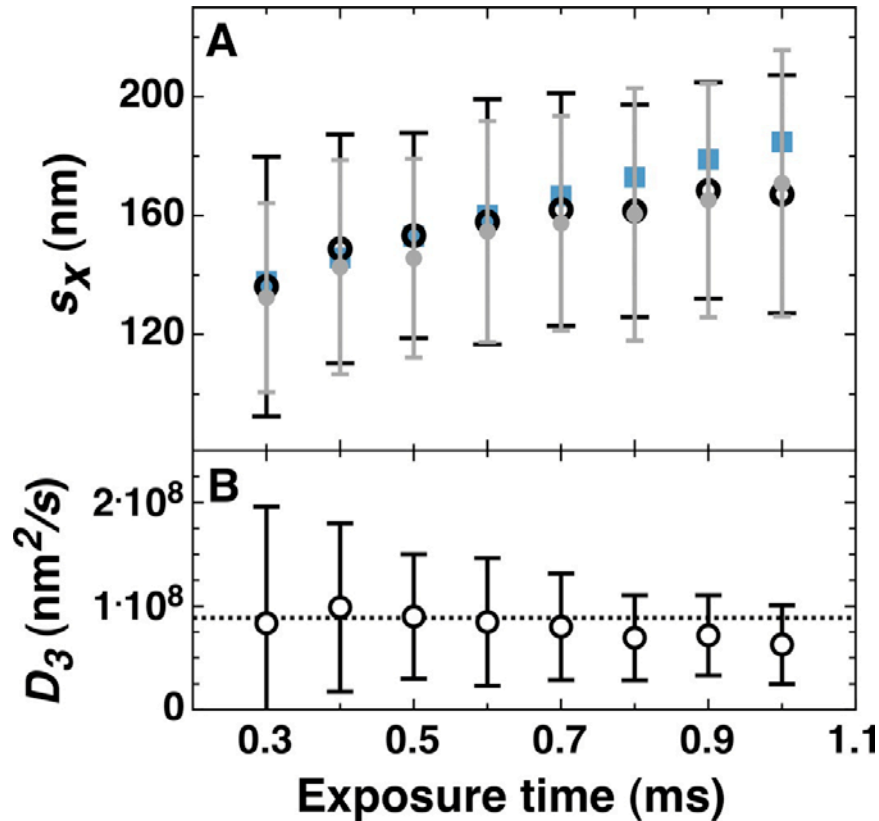


$$t = 0.3 \text{ ms} \Rightarrow \text{SD} = 137 \pm 28 \text{ nm}$$

$$t = 0.7 \text{ ms} \Rightarrow \text{SD} = 159 \pm 32 \text{ nm}$$

$$t = 1.0 \text{ ms} \Rightarrow \text{SD} = 172 \pm 35 \text{ nm}$$

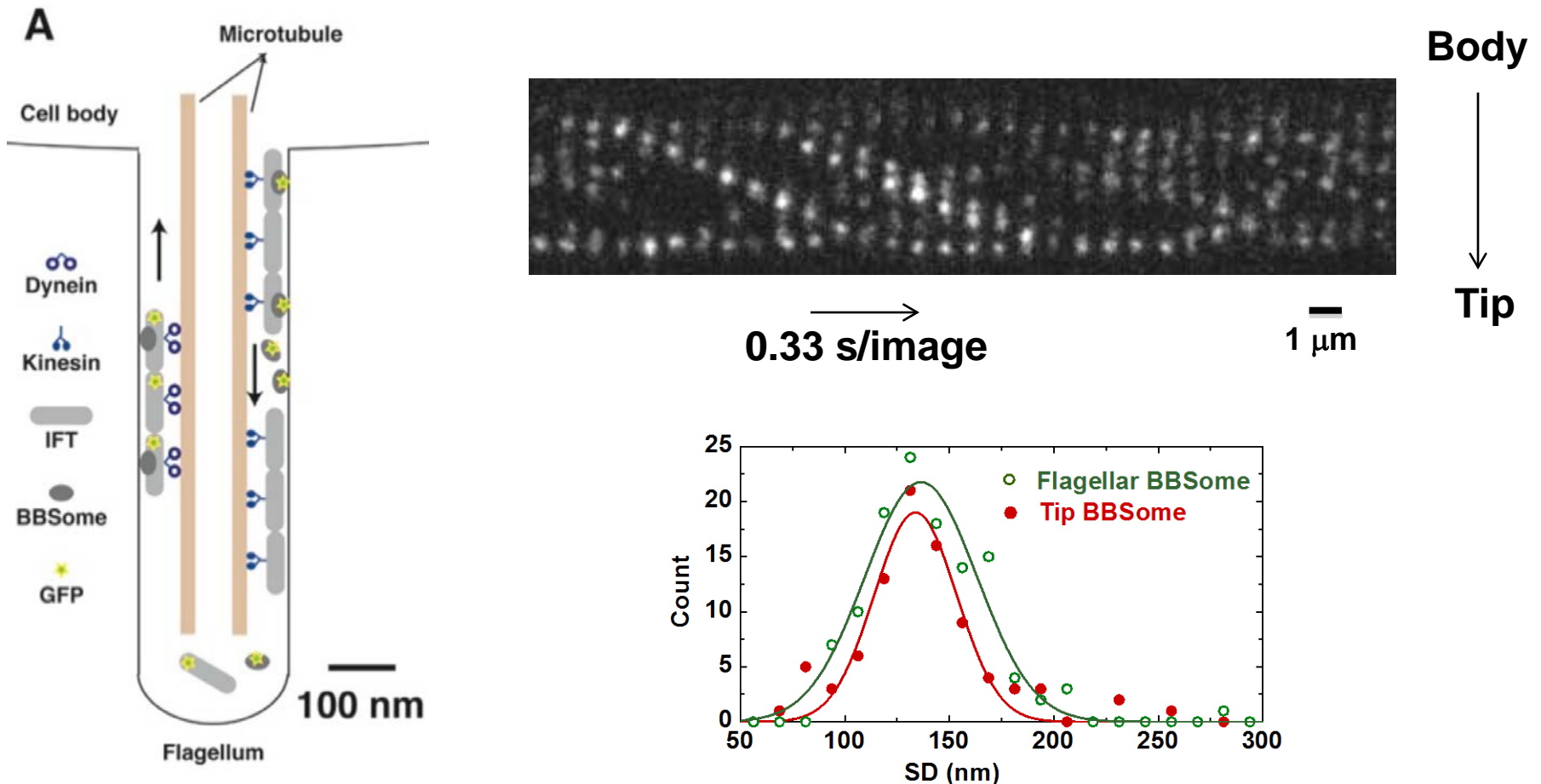
## SD vs. exposure time



For 1 GFP image at 1 ms,  $D_3 = 9 \times 10^7 \pm 5.2 \times 10^7 \text{ nm}^2/\text{s}$

For 10 images,  $D_3 = 9 \times 10^7 \pm 1.7 \times 10^7 \text{ nm}^2/\text{s}$

# Application III: BBSome and IFT turnaround mechanism at the flagellar tip



- If IFT dissociates at the tip, SD will increase by 5 nm for 1 ms exposure.
- We observe no increase.

# Summary

- **SIMA**, single-image molecular analysis, can speed up single-molecule fluorescence studies to millisecond timescales.
- **SMID**, single-molecule image deconvolution analysis, provides localization, separation, and dynamic information of single molecules with nanometer precisions.
- **Already show promise in biological systems**

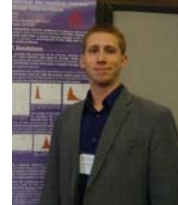
# Lab members



Shannon Zareh



Michael DeSantis



Jonathan Kessler



Anthony Kovacs

## Collaborators

IFT dynamics: **Susan Dutcher**, Department of Genetics, WU medical school

Photosynthesis: **Bob Blankenship**, Department of Biology, WU

UgtP dynamics: **Petra Levin**, Department of Biology, WU

# Imaging setup

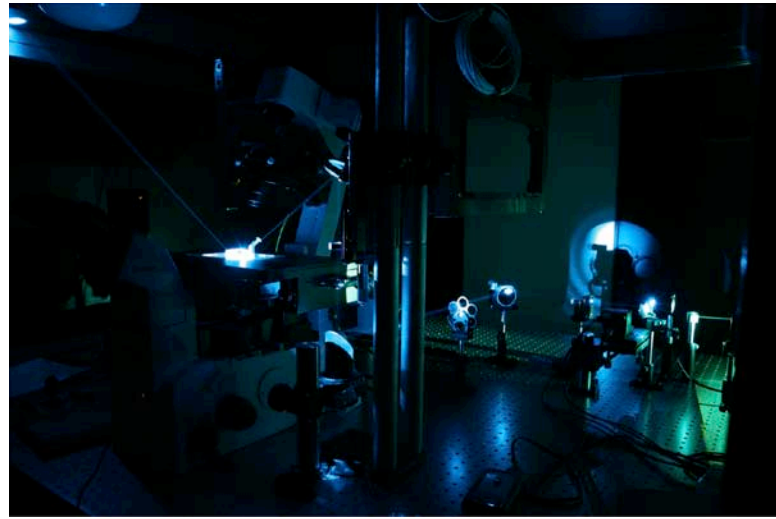
**Microscope imaging system**



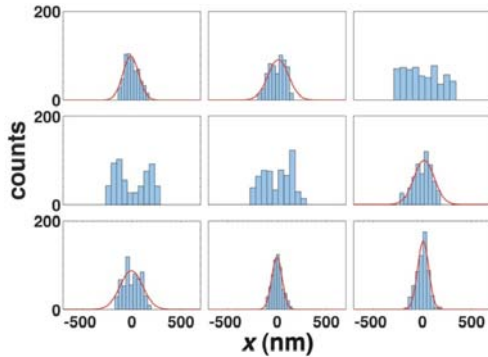
↑  
**Camera**

↑  
**Microscope**

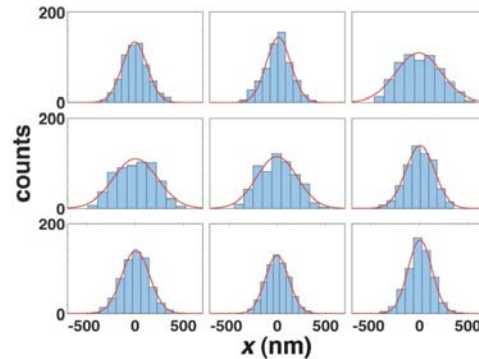
**488 nm laser and prism TIRF**



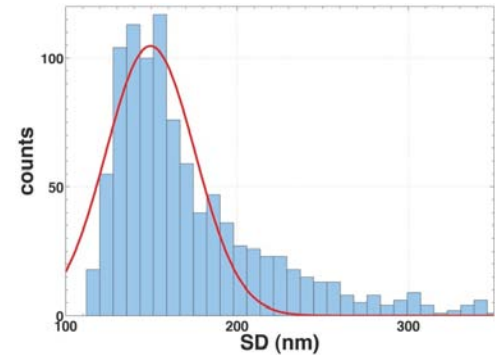
# PWDF<sub>xy</sub> can be approximated by a Gaussian



Single trajectory position distributions at 0.6 ms, PWDF<sub>xy</sub>



PWDF<sub>xy</sub> convolved with photon emission

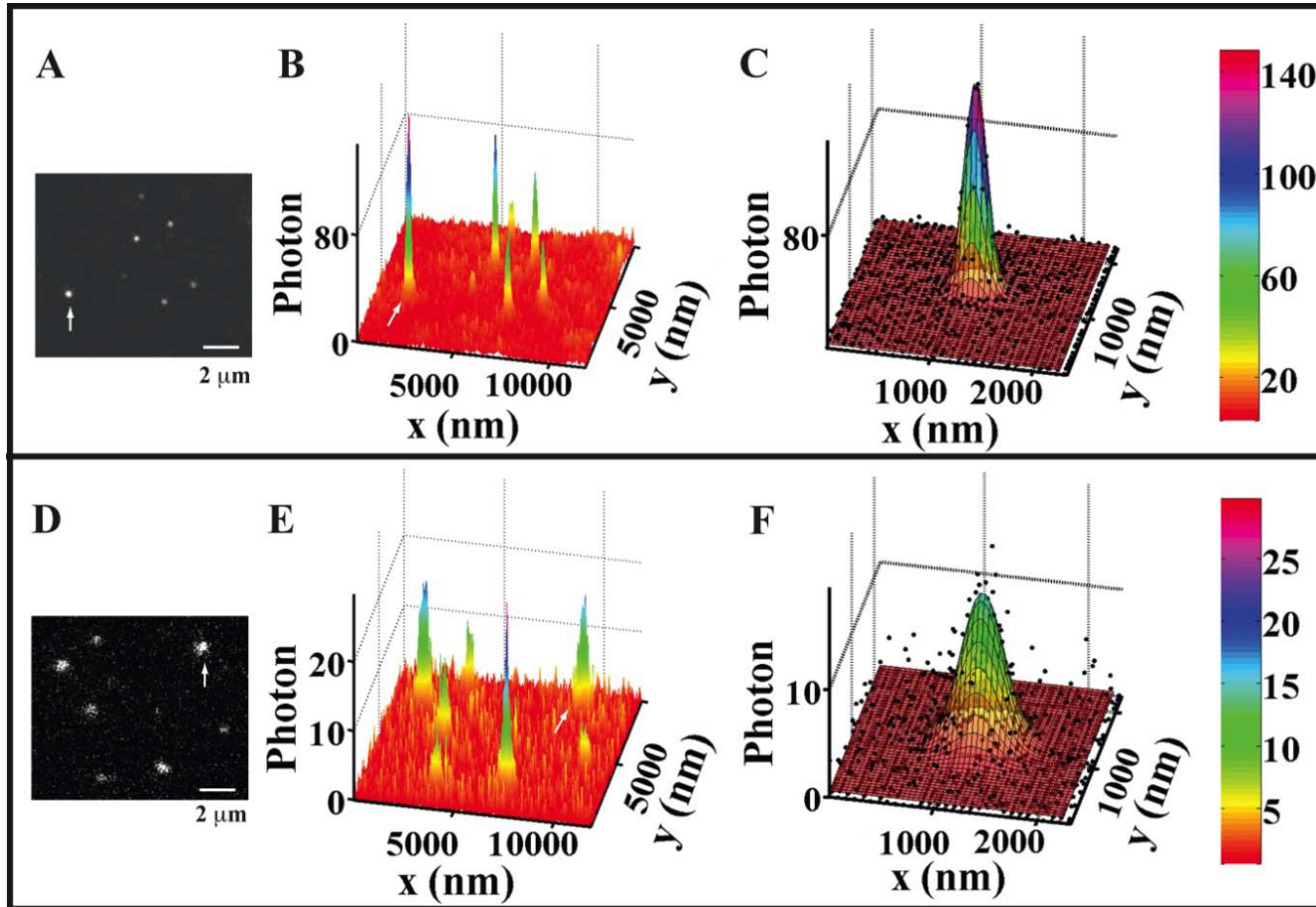


Convolved PWDF<sub>xy</sub> SD distribution; the mean yields  $A_{x,y} = 0.095$

$$PWDF_{x,y} = \frac{1}{2\pi A_{x,y} D_3 t} \exp\left(-\frac{x^2 + y^2}{2A_{x,y} D_3 t}\right)$$



# Immobile vs. diffusing GFP



# Questions:

1. Dynamics of BBSome in flagella to carry signaling proteins
2. BBSome, IFT, and Kinesin turnaround mechanisms at the flagellar tip

