Math 583 - Assignment #2 - Spring 1995

1. Consider the one-locus, two allele model with constant selection and no mutation, where x_t, the allele frequency of allele A at time t satisfies

$$x_{t+1} \;=\; F(\,x_t\,) \;=\; \frac{w_{11}\,x_t^2 + w_{12}\,x_t\,(1-x_t)}{W(\,x_t\,)}$$

and

$$W(x) \;=\; w_{11}\,x_t^2 + 2\,w_{12}\,x_t\,(1-x_t) + w_{22}\,(1-x_t)^2$$

Show that $F'(\,x\,) > 0$ for $0 < x \le 1$. Hint: rescale the function $F(x)$ by defining a new variable $y = \dfrac{1-x}{x}$.

2. For the one-locus, two allele model with constant selection and no mutation in problem #1, do a complete stability analysis for the dynamical system for x_t. That is show that (a) for the heterozygote inferior case the interior equilibrium x^* in (0,1) is unstable and that the boundary equilibria are locally stable, (b) for the heterozygote superior case the interior equilibrium x^* in (0,1) is globally stable, and (c) in the other cases, one of the boundary equilibria is globally stable.

3. Couple the equations in problem #1 to the equation for population size, N_t, given by

$$N_{t+1} \;=\; W(\,x_t\,)\,N_t$$

and consider the case of density-dependent selection in which the selection coefficients w_{ij} depend upon N_t. Suppose $w_{11} = R - r_{11}\,N_t$, $w_{12} = R - r_{12}\,N_t$, and $w_{22} = R - r_{22}\,N_t$, with these values changing at each time t as a function of N_t and $r_{ij} \ge 0$. Assume $R \ge 1$ (what happens if this assumption doesn't hold?).

(a) Suppose the population is fixed for one allele (say A) so $x_t = 1$. Describe the behavior of N_t.

(b) Using whatever calculation method you desire (e.g. calculator, Populus, MATLAB, etc.), carry out a number of simulations for the behavior of this system, until you can summarize the effect of varying the parameters, then write up this summary. Note that there are a number of ways to summarize the different cases, but please include in your handed in work at least a few graphs of N_t versus x_t.