Math 152 - Exam 2 - Spring 2005

You may use calculators for this exam. Do all your work on the sheets provided, not this problem sheet. Please use only one side of a sheet for your response, but be sure to show your work. Please put your name on each sheet and circle the final answer to each problem. Point values are in parentheses.

Honor Statement: By signing this statement I agree that I will not discuss any aspects of the material covered on this exam with any other individual until after 6:00PM on the day of the exam. Additionally, if anyone approaches me before 6:00PM requesting any information about the exam, I will report this individual's action to Dr. Gross.

Signature: _____ Section #: _____

1. Sketch the graphs of the following functions. State where all relative maxima and minima occur, give the location of any inflection points and state where the function is concave up and where it is concave down (12 pts each).

(*a*) $\qquad y = (x^3 - 1)^4$

(*b*) $\qquad y = 4\,x - 1 - \dfrac{1}{x}$

2. Solve the following differential equations subject to the given boundary conditions (12 pts each).

(*a*) $\qquad y'(t) = 2\,y(t) \qquad y(1) = 4$

(*b*) $\qquad y'(x) = 3\,x^2 + 2\,x - 1 \quad y(2) = 12$

3. For territorial animals, an advantage of maintaining a territory is the regular availability of food associated with the territory. However, as territory size increases, there are additional costs to the animal in order to defend the territory from intruders. Suppose that an individual which maintains a territory of area A (in hectares) must expend energy of 6 $A^{5/3}$ Kcalories/day and the food it obtains supplies it with 30 $A^{2/3}$ Kcalories/day.
(a) Write an equation for the net energy gain per day (its gain from the food minus its expenditure in maintaiining the territory) for an animal with territory size A hectares.
(b) What size territory should be maintained in order to maximize the net energy gain?
(c) What is the animal's maximum net energy gain per day? (12 pts.)

4. Find the most general antiderivatives of the following functions (10 pts each).

(*a*) $\qquad f(x) = x^3 - 2x^2 + 4x - 6$

(*b*) $\qquad g(t) = 2\cos(3t) + 4\sqrt{t} - \dfrac{1}{t}$

(*c*) $\qquad h(z) = e^{4z} + \dfrac{3}{z^{2/3}}$

5. Antibiotic drugs act to decrease the bacterial population of susceptible bacteria within an individual. Suppose that a patient is under antibacterial treatment via infusion in a hospital for a very serious bacterial disease, and the drug typically causes the patient's bacterial density to decline at a rate proportional to the current density of bacterial cells. This means there is a constant per capita death rate of bacteria once the antibiotic is applied. Suppose the initial bacterial density in the patient is 200 cells/ml and after one hour it has dropped to 60 cells/ml. (10 pts.)

(a) Find an equation for N(t), the bacterial cell density t hours after start of infusion of the antibiotic drug.

(b) At what time will bacterial density have dropped to 5 cells/ml?