Math151 at the University of Tennessee, Knoxville - Chat for October 19, 2015 with the course instructor, Louis Gross.

I will be online starting at 7:30PM and will be happy to answer questions regarding any aspect of the course, assignments, etc. You can type in this document to ask questions.

When you ask a question, please do not use your name because this document will be saved and publicly posted after we close it. I will be on-line at least until 8:30PM but will stay on longer if there are still questions. Note that I do not know the identity of anyone posting questions - each participant shows up as "Anonymous" animal.

Hello Lou, are you currently online?

Yes I am now online.

Ah great. After looking at the homework from 9.1 in the textbook and reviewing the exam, I had a couple of questions about eigenvalues.

In the textbook, the excercises 9.1 through 9.4 request that you find the dominant eigenvalue, and in part b state there is a normalized eigenvalue. What do each of there terms mean when you are working to find eigenvalues?

OK, first, there are at most two different eigenvalues for a 2x2 matrix. In each of the 9.1-9.4 these are 2x2 matrices so there are up to 2 eigenvalues. the dominant one is the one that is of largest (positive) value. For example the one we did in class today was
P=[1 4; .5 0] and we saw that ity had two eigenvalues - 2 and -1 so the dominant eigenvalue is 2. In general for all the cases we look at with Leslie Population matrixes, the dominant eigenvalue is the positive one of largest value - we could prove this but not in this class. The eigenvalue associated with this dominant eigenvalue gives the long-term fraction of the population in each stage class, once it is normalized to sum to one.

Ok then so for P=[0.7 0.6; .9 0] I would use the equation to find the two eigenvalues for the matrix, then identify which one has the largest positve value? Is that the right process?

Yes you would use the characteristic equation for the eigenvalues we gave in class today (in this case the Trace of the matrix is .7 and the determinant is -.54) and then find the two eigenvalues by solving this quadratic equation and then choose the largest one and find the eigenvector for it, normalize it so the entries sum to one.

Ok thank you, next is for the exam. question 1 asks us to identify whether the calculations are defined or undefined. With one of the questions, there are two matrices and then -3 in between them. I am pretty sure that this one is undefined as well as the one where the rows and columns between the two matrices do not match, such as 2x3 by 2x2. What conditions are defined to invalidate the equations we will be working on?

Good questions - the problem 1(a) is an appropriate operation because the 4 in front of the first matrix just means to multiply the matrix by 4 - so you just multiply each entry by 4 and then the 3 in front of the second matrix means multiply each entry of the matrix by 3. Then to solve it you check to make sure each matrix is of the same size since you can only add and subtract matrices of the same size. In this case both are 2x2 so you can then subtract them. Make sense?

Ah yes, thanks i see now. So the integer is viable in the equation because it is simply like when multiplying a equation in parentheses like 2(4x-20)?

Yes, this is why what we are doing is called matrix algebra - in many ways it is similar to the algebra you saw in high school but it does have some different rules - including multiplication (and division, though we haven't emphasized that).

Alright, thank you very much for the help. Thats all the questions i have for tonight, but I will hopefully have a few more for you on wednesday. Thanks again, good night

Could you explain how to find the eigenvectors for each of the eigenvalues for number 2 on the exam. I found the eigenvalues which are 3 and -1 but I'm a bit confused on how to use this information to find the eigenvectors.

OK - I'm going to try to use the math formatting to do this so give me a little time to try to get it to work

$$\begin{bmatrix} 2 & 6 \\ .5 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} x \\ y \end{bmatrix}$$

this is the equation to use to find the eigenvector associated with the eigenvalue 3 so you get an equation for x and y from this

2x + 6 y = 3 x   so x = 6y so if we choose y=1 then x = 6 and the eigenvector is $\begin{bmatrix} 6 \\ 1 \end{bmatrix}$

and we can normalize this to get

$$\begin{bmatrix} 6/7 \\ 1/7 \end{bmatrix}$$

now for the other eigenvector we substitute -1 where the 3 was above and we get

2 x + 6 y = -x   so 3x = -6 y or x = -2y and the eigenvector is $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

note that we cannot normalize a vector like this - it has negative entries.

OK?

Ok, so do we have to normalize the vector every time or unless stated?

There are vectors that cannot be normalized so you wouldn't try to do it for those. If the eigenvector has both positive and negative entries it cannot be normalized. For the population projection matrixes (and for the transfer matrix ones we saw first for examples such as ecological succession) all have a dominant eigenvalue which has an eigenvector with all positive entries so you can normalize it. But they can also have an eigenvalue (the example above has -1) with an eigenvector that cannot be normalized. For applications all we really are interested in is the dominant eigenvalue (which gives the long term growth rate and the eigenvector for this eigenvalue which gives the long-term fraction in each state when you normalize it.

Ok thank you for clarifying that for me.

No problem - anything else? If not I'm going offline.

That is all for tonight, thank you once again.