Math 151 – Sample Exam 2 – Fall 2015 – Louis Gross

It would be best if you try to take this Sample Exam as if you were sitting in class, using only a calculator. For the actual in-class exam, there will be blank sheets of paper handed out for you to give the answers but it will be important for you to SHOW YOUR WORK even if you are certain your answer is correct. Note that this Sample Exam is designed to be taken in the 60 minutes that you will have for the actual exam. You will not be allowed to keep the exam sheets.

The exam will have an honor statement similar to the below.

Honor Statement: By signing this statement I agree that I will not discuss any aspects of the material covered in this exam with any other individual until after 6:00PM Knoxville time on Thursday October 29. Additionally, if anyone approaches me before 6PM requesting any information regarding the exam, I will report this individuals' action to Dr. Gross.

Signature:____Section # __________

1. Carry out the following calculations where possible. If the calculation is not defined, clearly state that it is not defined. (5 pts each part)

(a) $4\begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix} - 3\begin{bmatrix} 1 & -1 \\ 4 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 & -1 \\ 4 & 0 & 1 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 2 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 6 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 4 \\ 5 & 2 & -4 \end{bmatrix}\begin{bmatrix} 3 & 2 & -1 \\ 4 & 0 & 1 \\ 6 & 5 & 3 \end{bmatrix}$

2. Find all eigenvalues and an associated eigenvector for each eigenvalue for the matrix (15 pts)

$$\begin{bmatrix} 2 & 6 \\ 1/2 & 0 \end{bmatrix}$$

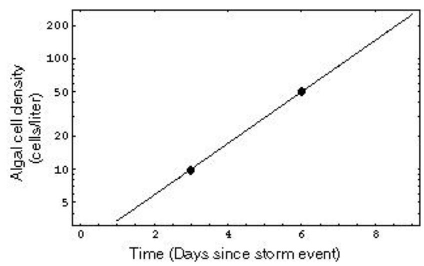
3. Biennial plants live for two years and have two stages (first year, F, and second year, S). For a population of this plant, each year on average only 10% of the F stage plants survive to become S stage plants and all S-stage plants die after the one year in this stage. F stage plants do not produce any offspring, while S stage plants produce on average 90 seeds which survive and germinate to become F stage plants the next year. (15 pts).

(a) Give a matrix equation for the number of F stage and S stage plants at time 1 year given that you know the numbers of F stage and S stage plants at year 0,,,,,,,,,. (4 pts)

(b) Suppose there were 10 F stage and 2 F stage individuals at year 0. How many of each stage would there be after 2 years? (2 pts)

(c) If this population were to live for a long time, what would the long term growth rate of the population be? If there were 1000 individuals present at time 100, how many individuals would be present at time 101 approximately? (6 pts)
(d) If this population were to live for a long time, what fraction would be F stage and what fraction would be S stage? (3 pts)

4. Below is a semilog graph showing data on algal density in a lake following a storm event that released a large amount of phosphorus into the lake. Note that the data fall on a line with two points at (3,10) and (6,50). (20 pts)
   a. Derive an equation for A(t) = the algal density on day t following the storm event (8 pts).
   b. Use your equation from (a) to find the algal density just before the storm event (that is at time 0) and to find the doubling time (how long it takes for the algal density to double). (8 pts)
   c. Estimate the time at which the algal density reaches 500 cells per liter (4 pts)



5. Matlab has the following population projection matrix defined:

```
>> P=[0 2 8; .2 0 0; 0 .5 0]
P =
        0    2.0000    8.0000
   0.2000         0         0
        0    0.5000         0
```

Describe in words what each of the following lines of Matlab commands produces (5 pts each line):

```
P*[10;0;0]
P^50*[10;0;0]
[v,e]=eig(P)
```

6. An epidemic of flu is spreading in a city, with individuals classified as being susceptible (S) to the flu, infected (I) , or removed (R) (a removed individual is one who does not have the disease and is not susceptible to getting it). The below graph shows the fraction of individuals in each state transferred per week to one of the other states (susceptible individuals become removed for some time if they get a flu shot). (15 pts)

(a) If $N_t = \begin{bmatrix} S \\ I \\ R \end{bmatrix}_t = \begin{bmatrix} Susceptibles \\ Infecteds \\ \mathrm{Re}\,moved \end{bmatrix}_t$ gives the number in each state in week t find the matrix P which described the transfer of individuals between each disease state from week to week according to $N_{t+1} = PN_t$ (6 pts)

(b) Suppose that there are initially 1000 susceptibles and 10 infected individuals. Find how many individuals there would be in each infection state after 1 week. (6 pts)

(c) Suppose the epidemic were to continue for a long time. Describe how you would find the long term fraction of individuals in each infection state. (3 pts). For EXTRA CREDIT (5 pts) calculate the long-term fraction of individuals in each state.