Math 152 - Sample Exam 3 - Spring 2005

Note that this Exam will cover sections 19-25 in the text but does rely on your knowledge of derivative rules covered in earlier exams so be sure you look over the earlier exams to be certain you understand how to do problems on them.

You may find the following formulas useful:

$$\int \frac{f'(x)}{f(x)}\, dx = \ln\left(f(x)\right) + C \qquad \int f'(g(x))\, g'(x)\, dx = f(g(x)) + C$$

$$\int f'(x)\, e^{f(x)}\, dx = e^{f(x)} + C \qquad \int f'(x)\; f(x)^n\, dx = \frac{f(x)^{n+1}}{n+1} + C \qquad \int u\, dv = u\, v - \int v\, du$$

1. The zooplankton density in a lake at noon is found to be reasonably well described by an exponential distribution since most of the zooplankton migrate towards the surface at mid-day. The density function at noon is

$$\rho_N(x) = 100\, e^{-x/2}$$

where x is the depth and $\rho(x)$ is the number of zooplankton per m^3 in a standard 1 m^2 water column. At midnight, the density function is

$$\rho_M(x) = 100\, x\, e^{-x/2}$$

Compare the total number of zooplankton in the water column down to the depth of 4 m at noon and midnight. At which time is there greater numbers of zooplankton down to this depth?

2. Find the most general antiderivatives of the following functions.

(a) $\quad f(x) = \sqrt{4x+1}$

(b) $\quad g(t) = \dfrac{t^4}{1+t^5}$

(c) $\quad h(y) = y^2\, e^{-y^3}$

(d) $\quad y(x) = x\, e^{-x}$

(e) $\quad f(x) = \dfrac{2}{x^2 - 2x - 3}$

(f) $\quad f(x) = x\, (x^2 + 4)^8$

3. Find the area under the curve $y = \dfrac{x}{1+x^2}$ from x = 0 to x = 2.

4. Find the area bounded betwen the graphs of $y = \sqrt{x}$ and $y = x^4$ for $x \geq 0$.

5. Find the volume of the solid of revolution generated by revolving the curve $y = x^2 + x$ about the x-axis for $0 \leq x \leq 1$.

6. A spherical tank has radius 4 meters. If it is half full of liquid of density 100 kg/m^3, how much work is done in pumping out all the liquid through a hole at the very top of the tank?

7. Find the following integrals:

$(a) \int_1^2 x \ln(x) dx$

$(b) \int_1^2 \frac{dx}{x\,(x+1)}$

$(c) \int_0^1 \frac{x}{\sqrt{2+x}}\, dx$