Math 152 – Spring 2016 – In-class problems– April 25, 2016.

Consider times at which a parent bird provisions nestlings – that is times between arrivals of the parent carrying food for the nestlings. Suppose that the waiting time between arrivals of a parent bird follows the exponential probability density function

$f(t) = .6e^{-.6t}$ for $t \geq 0$ and f(t)=0 for t<0

where the waiting times are in minutes.

(a) What is the mean waiting time?
(b) What is the probability that the wait for food between arrivals of a parent is between 2 and 3 minutes?
(c) What is the probability that the wait will be at least 3 minutes?

Given the differential equation

$$\frac{y'}{2+\cos x} - e^{-y} = 0 \qquad \text{with y(0)=0}$$

Find the solution y(x) and from this find $y(\frac{\pi}{2})$

A population of cells has growth rate (number of cells per hour) that is reasonably described by the differential equation

$$\frac{dN}{dt} = N(200-N)(N-40000)$$

(a) Find all equilibrium population sizes
(b) Determine for each equilibrium whether it is stable or not.

A patient initially enters a hospital with an initial blood concentration of a medication they have been taking of 50mg/l in their blood. Assume their blood volume is 5 liters. The hospital puts the patient on infusion of the same medication that has concentration of 400 mg/l and infuses the patient with this solution at a rate of .5 liters/hr. Assume that the patient eliminates

fluid from their blood at the same rate that fluid is being infused and that the concentration of the drug in the fluid eliminated is the same as the blood concentration at that time (e.g. the blood volume is well-mixed).

(a) Write a differential equation for x(t) = mg of drug in the patient at time t hours after entering the hospital.
(b) What is the equilibrium blood concentration of the drug if the patient is maintained on this schedule for a long time?
(c) Solve this differential equation for x(t)
(d) At what time will the drug concentration in the patient be twice what it was upon entering the hospital?