Math152 at the University of Tennessee, Knoxville - Chat for April 4, 2016 with the course instructor, Louis Gross.

$[\pi(\frac{x^5}{5}+\frac{x^4}{2}+\frac{x^3}{3}]$

I will be online starting at about 7:30PM and will be happy to answer questions regarding any aspect of the course, assignments, etc. though for this evening I suspect that questions will be mostly about Chapters 21 to 24 and the sample exam. You can type in this document to ask questions.

When you ask a question, please do not use your name because this document will be saved and publicly posted after we close it. I will be on-line at least until 8:30PM but will stay on longer if there are still questions. Note that I do not know the identity of anyone posting questions - each participant shows up as "Anonymous" animal.

I am online now - Lou

Dr. Gross, I was wondering if you could work out problems #4 and #7 on the practice exam? Thanks so much!

Thank you

For #4 we add the number of cases prior to day t=0 to the number of cases from day 0 to day 10 to get the total number of cases. To get the number of cases from day t=0 to day t=10 we integrate the rate of new cases C(t).
So the total number of cases is

$$100+\int_0^{10} C(t)\ dt$$

Then we need to find this integral to get the answer. To integrate this

$$\int_0^{10} 10t^2e^{-\frac{t}{5}}\ dt$$

We realize that this is a form we do not have an explicit way to find an antiderivative for. So we have only two choices - substitution and integration by parts. Since this is a product and one of the terms (t^2) gets a bit simpler when we take its derivative and the other term is easy to integrate, we use integration by parts letting u=t^2 and dv=e^(-t/5) dt

So with these choices for u and dv we get

$\frac{du}{dt} = 2t$ so $du = 2tdt$

And

$v = \int e^{-t/5}\, dt$ so using the formula for this antiderivative $v = -5e^{-t/5}$

And now using the integration by parts formula we get

$$\int 10t^2 e^{-\frac{t}{5}}\, dt = 10(t^2)(-5)e^{-\frac{t}{5}} - \int 10(-5)(2t)e^{-\frac{t}{5}}\, dt$$

And now we need to find the integral in this which is $\int 100te^{-\frac{t}{5}}\, dt$ but Note that again we can use integration by parts to find this using a similar choice of u and dv as used above so let u=t and dv = e^(-t/5)dt so that

$\frac{du}{dt} = 1$ and $du = dt$ and $v = \int e^{-t/5}\, dt$ so again $v = -5e^{-t/5}$

Then using the integration by parts formula again

$$\int 100te^{-\frac{t}{5}}\, dt = 100t(-5)e^{-\frac{t}{5}} - \int 100(-5)e^{-\frac{t}{5}}\, dt$$

So this is

$$\int 100te^{-\frac{t}{5}}\, dt = -500te^{-\frac{t}{5}} - 2500e^{-\frac{t}{5}}$$

And overall we get

$$\int 10t^2 e^{-\frac{t}{5}}\ dt = -50(t^2)e^{-\frac{t}{5}} - 500te^{-\frac{t}{5}} - 2500e^{-\frac{t}{5}}$$

Now we take this and evaluate it between the limits of t=0 and t=10 and this becomes 808 so when we add the initial 100 cases we get 908

OK?

Dr. Gross can you also do #5 please?

#5 This is the volume of a solid of revolution so we use the formula in the text to get

$$\int_0^1 \pi(x^2 + x)^2\ dx$$

And then calculate this by multiplying out the square to get

$$\int_0^1 \pi(x^4 + 2x^3 + x^2)\ dx$$

And then integrate this term by term to get

$[\pi(\frac{x^5}{5} + \frac{x^4}{2} + \frac{x^3}{3}]$ and take this between the limits of 0 and 1 to get

$$\pi(\frac{31}{30})$$

OK?

Can you please go over #3?

For #3 we realize that the area under a curve is just the integral of that function over the range of x-values so the area is

$$\int_0^2 \frac{x}{1+x^2}\ dx$$

So then we just need to find this integral. When we look at it, the top is almost the derivative of the bottom so we should try to substitute and see if that helps. So we let

$u = 1 + x^2$ and so $\frac{du}{dx} = 2x$ so $du = 2xdx$ and so $(1/2)du = xdx$

So when we substitute we get

$\int \frac{x}{1+x^2}\, dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}ln(u)$ so

$$\int \frac{x}{1+x^2}\, dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}ln(u) = \frac{1}{2}ln(1+x^2)$$

And then we plug in the limits from 0 to 2 to get $\frac{1}{2}(ln(5) - ln(1)) = \frac{1}{2}ln(5)$

Could you please explain how to do #2a and 2b?

For #2(a) you just use the power rule for antiderivatives and take the antiderivative term by term - the same is true for #2(b) which I did at the end of class today.

Could you please answer #7?

For #7 we need to take the integral of the temperature T(t) from t=0 to t= 12 and then divide by 12. This gives the average monthly temperature. You should be able to guess what it is by realizing that the T(t) is a constant (14) plus a sin function which has period of 12 months.

So the average temperature is

$$\frac{1}{12}\int_0^{12} T(t)dt$$

And we plug in the formula for T(t) in here to get

$\frac{1}{12}\int_0^{12} (14 + 12sin(\frac{\pi}{6}(t-4))dt$ for the first part of this we get

$\frac{1}{12}\int_0^{12} 14\ dt = 14$ and for the second part we can substitute

$u = \frac{\pi}{6}(t-4))$ so $du = \frac{\pi}{6}dt$ and then the second part becomes (note that we get 12/12 in front so this cancels and we have $\int \frac{6}{\pi}sin(u)du = -\frac{6}{\pi}cos(u)$ so in the original t variables this is $-\frac{6}{\pi}cos(\frac{\pi}{6}(t-4))$ and we take this and evaluate it between the limits of t=0 and t=12 - you see that it =0 so the average temperature is 14

OK?

Could you please work out #6b and c?

For #6b just do a substitution letting u=2x+4 so du=2 dx and substitute it in to get $\int \frac{1}{2}u^{-1/2}du = \frac{1}{2}(2)u^{1/2} = u^{1/2}$ and then go back to the original x values and take the limit from x=0 to x=1

OK?

For #6c, the integral has a product so we try to use integration by parts - let u=x and dv=sin(2x) dx and then use the integration by parts formula. Let me know if you want me to go through it in detail

Could you explain #1a and b?

I did this in class for the case of the zooplankton density at noon and the exact same method is applied for the midnight case except that the integral is easier - you don't need to use integration by parts. Note that the answer to (b) is found just by comparing teh two answers you get for (a).

If you had asked how to find the depth of maximum density on 1A, how would you do that? Thanks

Good question - I did the case of what happens at noon in class today - we set $\rho'(x) = 0$ and the case of what happens at midnight is interesting because when you look at the function $\rho(x) = 1000e^{-x/2}$ you see that when you take its derivative it is always negative - it is a decreasing function - so the maximum value is when x=0 - the density is highest at midnight right near the water surface.

On the sample exam answers you for #7, you have the integral going from 12 to 0, however if you let 0=Jan, 1- Feb, 2-Mar, 3-Apr, 4-May, 5-June, 6-July, 7-Aug, 8-Sep, 9-Oct, 10-Nov, and 11-Dec, wouldn't the integral just be from 11 to 0 instead of 12 to 0? Thanks

You have pointed out an issue when you use continuous variables (in this case time t) for times that are inherently discrete (months). The confusion arises because the integral has to cover a period of length 12 months and 12 - 0=12 so the integral must be from 0 to 12. The problem arises in terms of what for example a 1 would mean. If we took the integral from time t=0 to time t=1 that clearly covers one month and goes from Jan 1 to Feb 1. The 12 covers the period from Dec 1 to Jan1 of the next year.

That makes sense, thanks!

If there isn't anything else - I am going to go offline - I wish you all the best - try to get a good night's sleep.