**Math 152 – Sample Final Exam – Spring 2016 – Louis Gross**

It would be best if you try to take this Sample Exam as if you were sitting in class, using only a calculator. For the actual in-class exam, there will be blank sheets of paper handed out for you to give the answers but it will be important for you to SHOW YOUR WORK even if you are certain your answer is correct. Note that this Sample Exam is a bit longer than the actual exam will be to give you additional practice. On the actual exam the formulas for derivatives and antiderivatives below will be provided, but I expect that you will know all the other rules for finding derivatives and antiderivatives. This exam covers all parts of the course with an emphasis on Chapters 25-27 which have not been covered on previous exams.

Table of Derivatives and Antiderivatives:

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$D_x(mx+b)=m$ $\quad D_x(Ce^{kx})=Cke^{kx}$ $\quad D_x(Ca^x)=\ln(a)Ca^x$

$D_x(\sin(Cx))=C\cos(Cx)$ $\quad D_x(x^n)=nx^{n-1}\ for\ n\neq -1$ $\quad D_x(\ln(x))=\frac{1}{x}$

$D_x(\cos(Cx))=-C\sin(Cx)$

$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ for\ n\neq -1$ $\quad \int \frac{1}{x}\,dx=\ln(x)+c$ $\quad \int \sin(x)\,dx=-\cos(x)+c$

$\int \cos(x)\,dx=\sin(x)+c$ $\quad \int e^{ax}dx=\frac{1}{a}e^{ax}+c$ $\quad \int a^x dx=\frac{a^x}{\ln(a)}+c$

$\int \frac{f'(x)}{f(x)}\,dx=\ln(f(x))+c$ $\quad \int f'(g(x))g'(x)\,dx=f(g(x))+c$

$\int f'(x)e^{f(x)}dx=e^{f(x)}+c$ $\quad \int f'(x)f(x)^n dx=\frac{1}{n+1}f(x)^{n+1}+c$

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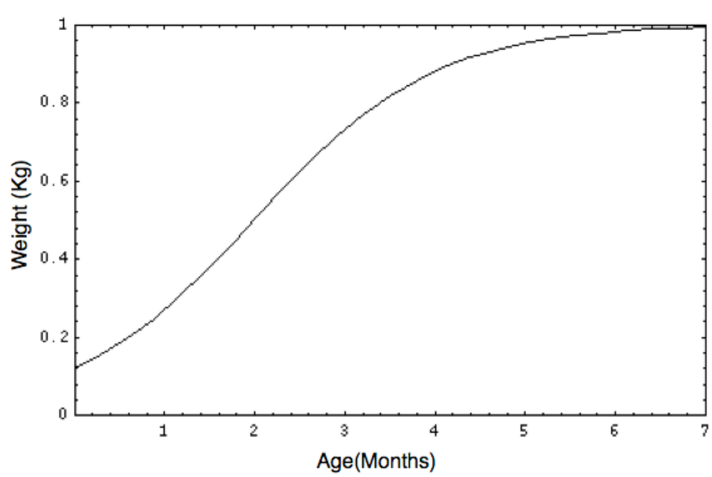
1. The average growth in weight of an individual fish is shown in the below graph.



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Suppose W(a) gives the fish weight in kg at age a months where $0 \le a \le 7$
(a) Define $W'(a)$ and state its units.
(b) Estimate $W'(2)$.
(c) Sketch a graph of how (approximately) $W'(a)$ changes from age zero through the end of age 6 months.

2. The length of a fish of age t grows according to $\dfrac{dL}{dt} = K(L_\infty - L)$ so

$$L(t) = L_\infty(1 - e^{-K(t-t_0)})$$

Assume $L_\infty = 40$ cm , K = .2/month and L(0)=3 cm.

(a) How fast is the fish growing in length at birth?
(b) At what age will the fish reach 1/2 of its largest length?

3. Find the solution of the differential equation $\dfrac{dy}{dt} = (2t+1)y$ if y(0)=4.

4. Suppose a random variable X has probability density function
$g(x) = \alpha + x$ for $0 \le x \le 1$ with g(x)=0 elsewhere.
(a) Find the value of $\alpha$ which makes g(x) a probability density function.
(b) Find $P[X \le 1/2]$
(c ) Find the expected value of X, E[X].
(d) Find the median of X.

5. A small lake holds 20,000 liters of water. Starting at time t=0, a chemical plant releases water containing .02 kg per liter of phosphate into the lake at a rate of 100 liters per hour. The lake is well mixed and water is leaving the lake at a rate of 100 liters per hour so that the volume of the water in the lake stays constant. The concentration of phosphate in the lake at time t=0 was .005 kg per liter.

(a) Write a differential equation for x(t) = how much phosphate (in kg) is in the lake at time t hours.
(b) How many kg of phosphate were in the lake at time t=0?
(c) Solve the differential equation in (a) and use the information in (b) to find an equation for x(t).
(d) At what time will the amount of phosphate in the lake be twice the amount present at time t=0?

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6. A model for the density of cells N(t) at time t in a culture follows the differential equation

$$\frac{dN}{dt} = \frac{2N}{1+N^2} - N$$

Find all biologically-realistic equilibria for this model and determine whether each one os stable or not.

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7. Find the area bounded between the graphs of $y = 2x^2$ and $y = 4x$

8. The vertical probability density function for a marine invertebrate in the water column in the ocean is

$f(x) = 6xe^{-3x^2}$ where x is the depth in meters.

(a) What depth gives the highest probabilty density of invertebrates?

(b) What fraction of invertebrates are between the top of the ocean and 1 meter depth?

9. As a laboratory assignment in a biology class, you are asked to make observations on a fungal culture in a petri dish to test the hypothesis that the growth rate of the culture is proportional to the culture's size. How could you do this assuming that the easiest observation you can make is the area of the fungal colony? Give a condition under which you would conclude this hypothesis is refuted.

10. Find the following

(a) $\int 3xe^{-4x}dx$ (b) $\int_0^{1/2}(4x+\sin(2\pi x))dx$

11. Find the following

(a) $D_t y(t)$ if $y(t) = 4t\ln(2t+1)$

(b) $g'(x)$ if $g(x) = \frac{x}{x+1}$

12. The above ground biomass of a uniform age stand of trees is given by

$B(a) = \frac{1500}{10+90e^{-a/10}}$ tons/hectare where a is the stand age in years.

(a) What is K = the long-term biomass in this stand?

(b) What is the growth rate of the stand biomass?

(c) At what age will the stand biomass be 1/2 of the long-term biomass (e.g. find a so that B(a) = K/2)

(d) It can be shown that $B'(a) = rB(\frac{K-B}{K})$ where r =1/10 and K is the value from (a).

Show that $B'(a)$ is maximized when $B(a) = \frac{K}{2}$